A Study on Comet ISON's Orbit

Project submitted in partial fulfillment of the requirements for the award of the degree of

Bachelor of Science In Physics

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2011-2014

Certificate

This is to certify that the project entitled, 'A study on Comet ISON's Orbit' is an authenticated record of work carried out by Sreedevi Varma.N., Reg.No.11122695 in partial fulfillment of the requirement for the award of the Degree of Bachelor of Science in Physics which is submitted to Mahathma Gandhi University Kottayam, during the period 2011-2014.

Dr.N.Shaji
Project Guide

MAHARAJA'S COLLEGE ERNAKULAM

Bachelor of Science In Physics

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Abstract

C/2012 S1 or Comet ISON, also known as Comet Nevski- Novichonok was a sungrazing comet discovered on 21 September 2012 by Vitali Nevski and Artyon Novichonok. The discovery was announced by the Minos Planet Centre on 24^{th} September.
This is an investigatory project to study about the orbit of Comet ISON or C/2012 S1. C/2012 S1 implies, a comet found in 2012 on the first of September. The project aims to find the position of the comet before and after perihelion, with perihelion distance and some other position and velocity.Graphs are drawn using Origin 6.1 and the report is prepared using LaTex

Acknowledgement

Let me avail this opportunity to express my deep sense of gratitude and sincere thanks to all those helped me to complete the project work sucessfully. I would like to express my thanks to Dr.N.Shaji, Associate Professor, Department of Physics, Maharajas College for his immense help, sincere guidance and timely suggestions during this project. I thank Dr.V.J.Dann, Assistant Professor, Department of Physics, Maharajas College for his constant support.

I would like to record my heartful thanks to all teachers of Physics Department for the support extended to me in all times of need. Finally, I wish to express my deep thanks to my friends, for their timely assistance and cheerful co-operation.

Above all, I thank my parents and the God Almighty for their blessings which helped in the successful completion of the project.

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1 Introduction

"The most beautiful thing we can experience is the mysterious. It is the source of all true art and science. He to whom the emotion is a stranger, who can no longer pause to wonder and stand rapt in awe, is as good as dead —his eyes are closed."- as said by Albert Einstein. The night sky has always fascinated us. Her beauty is so mysterious that everyone loves her.

The study of these mysterious behaviors lead to the opening of the greatest of all sciences, the Astronomy, the science of stars. It does not just study about the stars, but everything that comes within our eyes and beyond belongs to astronomy.

A special entity among them is the comet. The one with long tail, the one which showers us all. There is a lot of mystery around them. Their study may lead to origin of life or even origin of the Solar System. These planetesimals are also bound with Astrology, the branch of faith. Some believe that the shooting stars are lucky and they are nothing less than evil for some others. Whatever they may be, it is very fascinating to study about them.

Here is an investigatory project to study about the orbit of a newly discovered comet called Comet ISON or C/2012 S1, also known as Comet Nevski-Novichnok, after the founders. C/2012 S1 implies, a comet found in 2012 on the first of September. The project aims to find the position of the comet before and after perihelion, with perihelion distance and some other position and velocity.

Comets are merely icy planetesimals- Comet nuclei – a drift in the frigid out reaches of the Solar system. Comet nuclei put on a show only when they come deep enough into the inner solar system to suffer destructive heating from the sun. When they are close enough to show the effect of solar heating we call them active comets. Comet nuclei spend most of their lives as nothing more than icy-planetesimals-small dumps of primordial material.

The comet has almost a parabolic orbit, accurately it is hyperbolic. C/2012 S1 will come to perihelion on 28^th November 2013 at a distance of 0.01244 AU. Around that day the comet is expected to go around an angle of 240^0 around sun, with a velocity approximately 380 km/s. By August 27, 2014 the comet will be at a distance approximately 4.6 AU from Sun. Since the orbit is nearly parabolic the comet shall never come back again. Also the inclination of 62.39^0 from solar plane, the comet might be from the Oort cloud. At perihelion the temperature may reach high enough to melt iron. Also, it will come within th Roche limit , meaning it may disintegrate due to Sun's gravity. The faith might be similar to Shoemaker-Levy 9

The stories of comets are more fascinating and interesting. These hairy ones are more than a visual treat.

2 Solar System and it's Physics

2.1 History of Solar System

In these early years of 21^{st} century we have come to see our solar system as an unmistakable byproduct of the birth of the sun, rather than as a random collection of planets and moons. It is this understanding that brings order to what we see around us today. The solar system was formed from a rotating disk of gas and dust. Roughly 5 billion years ago, the newly formed sun was drifted in the interstellar space. The sun was not yet a star in the true sense of the word because the nuclear fires that power sun today had not yet started to ignite. It was a protostar – a large hot ball of gas that shone due to gravitational energy being turned into thermal energy and radiation, as the protostar collapsed from a cloud of interstellar gas. Back then the sun was a flat , rotating disk of gas and dust called protostellar disk. The protostellar disk capable of producing planets is called protoplanetary disk. It probably contained less than 1% as much mass as the nascent stars at the centre, but this amount was more than enough to account for the bodies that make up the solar system today. Instead of falling in the centre star the cloud collapsed into a disc. And it was the universal force called gravity which helped the planetesimals to grow into planets. This twisting cloud could also account for some directional orbits.

As the planets formed, they formed with a primary atmosphere. Massive planets still have their primary atmosphere while the less massive planets lost their primary atmosphere and then formed secondary atmosphere. Rocky terrestrial planet formed inside inner solar system. The giant planet in the outer solar system formed cores from planetesimals and captured gaseous hydrogen and helium onto their cores. Moons formed from mini – accretion disks around the planets. Not all planetesimals in the protoplanetary disk went on to become planets. Jupiter is a true giant planet. Its gravity kept the region of space between it and Mars so "stirred up", that most planetesimals there never coalesced into a single planet. This region, now referred as the asteroid belt , contains many planetesimals that remains from this rearely time. In the outer most part of the Solar system planetesimals also persist to this day. Born in a "deep freeze" , these objects retained most of the highly volatile materials found in the grains present in the proto planetary disk at the time of formation. Icy planetesimals in the outer solar system remain today as Comet nuclei – relatively pristine samples of the material from which our planetary system formed. Frozen Pluto and the distant dwarf planet Eris are especially large examples of these denizens of the outer solar system.

The implications of this conclusion are profound. Planets are a common byproduct of star formation. So, in a galaxy of a hundred billion stars, and a universe of hundreds of billions of galaxies , how many Earth-like planets might exist? It was just a large cloud. Now it host life. Who knows what comes next?

2.2 Gravity and Orbit

Gravity is a force between any two objects due to their masses. A body falling towards earth is accelerated in response to the force of gravity. The gravity obey what is called an inverse square law. This means that the force of gravity is inversely proportional to the square of distance between them(two objects) or

$$F \propto \frac{1}{r^2} \tag{1}$$

Newtons universal law of gravitation, states that gravity is a force between any two objects and has the properties

1. It is an attractive force acting along a straight line between two objects
2. It is proportional to the mass of one object times the mass of the other object
$$F \propto \ m_1 m_2 \tag{2}$$
3. It decreases in proportion to 1 divided by the square of the distance between the two objects
$$F \propto \frac{1}{r^2} \tag{3}$$

Written as a mathematical formula , the universal law of gravitation states that

$$F = G \times \ \frac{m_1 m_2}{r^2} \tag{4}$$

This force has its hand in the motion of each and every particle in and out the Solar system. One special kind of motion that arises from this force is the orbital motion. Before that we need to know what is an orbit. An orbit is : one body freely falling around another. Orbiting means falling around the world. Weightlessness is an effect due to such a free fall. When one object is falling around another, we say that the less massive object is a satellite of the more massive object.

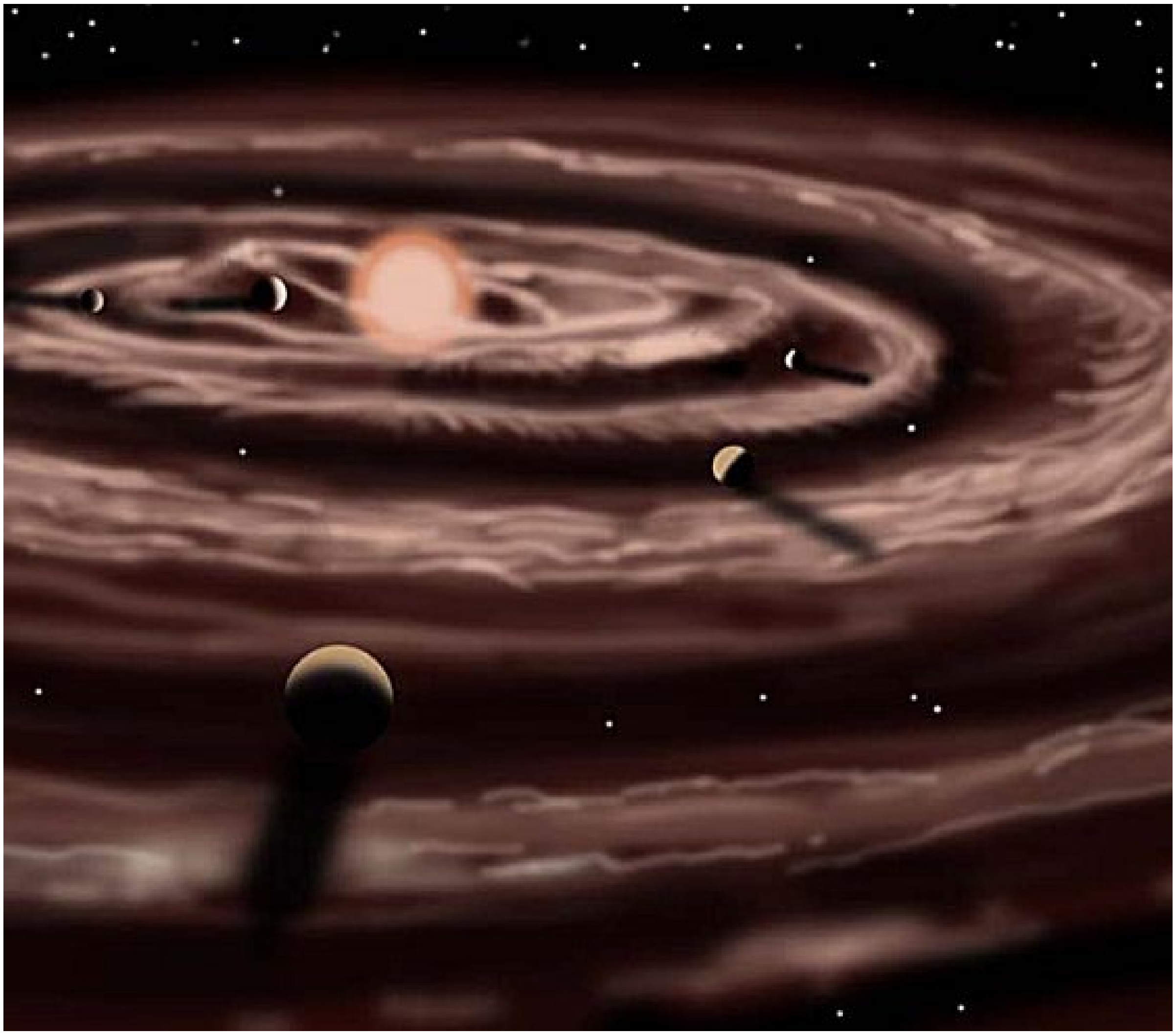

Figure 1: Protostellar Sun

In that case all bodies must fall down. But it doesn't happen. That means there is a constant supply of energy or force to maintain a circular motion. This force is called Centripetal force. In nature gravity provides the centripetal force that holds a satellite in its orbit. The velocity that is required for circular motion can be found as

$$v_{cir} = \sqrt{\frac{GM}{r}} \tag{5}$$

If the velocity is below v_{circ} the object will fall down and if it is above it , the object will escape from the gravitational force of the central body. That velocity is known as escape velocity. If we were to work through the calculations, we would find that the escape velocity is a factor of $\sqrt{2}$ times the escape velocity.

$$v_{esc} = \sqrt{\frac{2GM}{r}} = \sqrt{2}\ v_{cir} \tag{6}$$

But if the satellite velocity is less than v_{esc}, but greater than v_{cir}, the satellite will have an orbit which is elliptical. We call such orbits as bound orbits and unbound orbits are orbits which have their velocities greater than v_{esc}. Unbound orbits are either parabolic or hyperbolic. If the velocity is equal to v_{esc}, the orbit is strictly parabolic.

2.3 Gravity and Kepler′s laws

Nature has always been a wonder and a source of inspiration for intellectual activity to man. Apart from the physical components of the universe, there are the natural phenomena and a whole variety of rules and mechanisms that are responsible for the entire universe to function, as a simple system. It is a fact that phenomena like natural cycles, planetary movements and a host of other occur with clockwise precision and accuracy. The motion of the planets is mathematically describable and is governed by the following three laws which are due to Kepler. The first two laws were published in the year 1609 , while the third one in the year 1619.

First Law: Every planet revolve in an elliptical orbit with sun as one of its foci. Mathematically it is,

$$r = \frac{p}{1 + ecosr} \tag{7}$$

Second Law: Every planet revolves in its orbit in such a way that the line joining it to the sun sweeps out equal areas in equal intervals of time. That is, areal velocity is a constant.

$$\frac{dA}{dt} = \frac{1}{2}\ r^2\dot{\theta}\hat{e_A} = aconstant = \frac{1}{2}\ h\hat{e_A} \tag{8}$$

$$\implies r^2\dot{\theta} = h \tag{9}$$

Differential equation of the orbit can be derived from the fact that angular momentum and the total energy of the system are constants.

Newton' s law of universal gravitation is simple by itself. Yet when applied to extended rather than point like masses or when acting among three or more objects, this simple rule give rise to a surprising diversity of phenomena. The Kepler's law itself are certain examples. But doesn' t mean that gravity is just these laws. Let us see some examples.

1. Symmetries that allow us to say a great deal about the gravity of an object without actually calculating anything.

2. That the gravity within a spherical object is determined only by the mass within the radius

3. Tides on the Earth resulting from the fact that gravity from the moon and sun pulls harder on one side of the Earth than other.

4. Tidal interaction between planets and moon' s that locks a moon' s rotation to its orbit.

5. Comets that have been shattered by tides, and tortured moon' s alive with tide-powered volcanism.

6. Orbital resonance that nudge asteroids from their orbits and sweep out gaps in planetary rings.

7. Chaotic , unpredictable orbits in which the tiniest difference at one moment in time leads to huge difference later on.

2.4 Asteroids , Meteorities and Comets

Asteroids and Comets are tiny in comparison to planets; Yet their objects and their fragments that fall to Earth as meteorities, have told much of what we know about the early history of Solar system. As we explore these bit of interplanetary flotsam and jetsam, we will discover

1. Small, irregular world called asteroids that are made of rocks and metal.

2. That some asteroids are primitive while others are differentiated or are pieces of larger, differential bodies.
3. Different types of meteorites that are fragments of these varieties of asteroids.
4. That most asteroids orbit between Mars and Jupiter, but some have orbit that cross Earth' s.
5. Comet nuclei-pristine, icy planetesimals- a drift in the frozen outer reaches of the Solar system.
6. Spectacular active comets that warmed by the Sun as they dive through the inner Solar system
7. Meteor showers that occur when Earth passes through a Comet' s tail on debris
8. World-jarring meteorite impacts that still occur today and have played a vital role in shaping the history of life on Earth

3 Comets

3.1 Comets, Kuiper Belt and Oort cloud

Comets are merely icy planetesimals- Comet nuclei– a drift in the frigid out reaches of the Solar system. Comet nuclei put on a show only when they come deep enough into the inner solar system to suffer destructive heating from the sun. When they are close enough to show the effect of solar heating we call them active comets. Comet nuclei spend most oftheir lives as nothing more than icy-planetesimals-small dumps of primordial material.

We know where comets reside by observing their orbits as they pass through the inner solar system. Bored on these studies they seem to fall into two distinct groups: the Kupier Belt comets and Oort cloud comets. These two populations of comet nuclei are named for scientists Gerard Kupier and Jan Oort.

Comet nuclei from the Kuiper Belt orbit the sun within disk shaped region that begins just beyond the orbit of Neptune and extends outward to perhaps a thousand astronomical units from the sun. The innermost part of Kuiper Belt which is to end somewhat abruptly at about so AU, appears to contain tens of thousands of icy planetisimals, which we call Kuiper Belt Objects (KBOs).

Unlike the flat disk of the Kuiper belt, the Oort Cloud is a spherical distribution of the comet nuclei that are much remote to be seen by even the most powerful telescope. We know the size and shape of the Oort cloud because comet nuclei from the Oort Cloud approach the inner solar system from seemingly random directions in the sky, following orbits that bring them in from as far as 50,000 AU from the sun or about a fifth of the way to the nearest stars.

We can think Kupier Belt and Oort Cloud as enormous reservoirs of icy planetary material that now and then fall into the inner solar system.

3.2 Anatomy of an Active Comet

The small object at the centre of the comet is the comet nucleus. The nucleus is by far the smallest component of a comet, but it is the source of all the mass that we are stretched across the skies as the comet nears the sun. Comet nuclei are anywhere from a few dozen meters to several hundred kilometers across. We might formally describe comet nuclei as "planetesimals of modest size composed of mixture of various ices of volatile compounds, organics and dust grains loosely packed together to form a porous conglomerate". In the middle of the 20^{th} century astronomer Fred Whipple offered an elegant way to sum allof this up in just words; comets are "dirty snow balls"

As a comet nucleus nears the sun, sunlight heats its surfaces, turning volatile ices into gases, which then stream away from the nucleus, carrying embedded dust particles along with them. This process of conversion from solid to gas is called sublimation.

The gas and dust driven from the nucleus of an active comet form a nearly spherical atmosphere cloud around the nucleus called the coma (plural form: comae). The nucleus and the inner part of the coma are sometimes referred to collectively as the comet's head. Pointing from the head of the comet in a direction more or less away from the sun are long streams of dust, gas and ions called the tail.

The tail which is the largest and most spectacular part of a comet, is also the "hair" for which comets are named. Comets comes from the Greek word Komets, which means "hairy ones".

3.3 Sun and Comets. Comets and its tail

The tai, which is the largest and most spectacular part of a comet, is also the "hair " for which comets are named. Comet comes from the Greek word Komets, which means "hairy ones". Active comets have two different types of tails. One type of tail is called the ion tail. Many of the atoms and molecules that make up a comet' s coma are ions. Because they are electrically charged, ions in the coma feel the effect of the solar wind, the stream of charged particles that blows continuously away from the sun. The solar wind pushes on these ions, rapidly accelerated to a speed of more than 100 km/s – far greater than orbital velocity of the comet itself and sweeps them out into a long wispy structure. Because the particles that make up the ion tail are so quickly picked up by the solar wind, ion tails are usually very straight and point directly away from sun.

Dust particles in Coma can also have a net electric charge and force of the solar wind. In addition, sunlight itself exerts a force on cometary dust. But dust particles are much more massive than individual ions, so they are accelerated more gently and do not reach such high relative speed as the ions. As a result, the dust particles are unable to keep up with the comet, and the dust tail often appears to gently curve away from the head of the comet as the dust particles are gradually pushed from the comet's orbit in the direction away from the sun. Both type of tail always point away from the sun independent of the direction of the comet. On approaching sun the tail will be behind and on going back tail will be in front.

By electromagnetic theory this radiational pressure can be equated as

$$p = \frac{s}{c}\ for\ absorbing\ p = \frac{2s}{c}\ for\ reflecting \tag{10}$$

Thus p will lie between these two values.

3.4 Orbital Motion, Orbits of Comets

Orbital motion and gravity is already treated under "Gravity and Orbits". Before we enter the detailed study of orbits in general, let us stick to the topic's most important area, the Orbits of Comets.

How long a dying nucleus will survive depends on its orbital period and perihelion distance. Each passage takes its toll of ice and dust. By convention, comet orbits are generally referred to as short-period or long comet orbits. The division between the two is somewhat arbitrarily set at a period of 200 Earth years. Comets with periods of less than 200 years are called short-period comets. Comets with orbital period longer than 200 years are termed as long period comets.

When a comet nucleus first enters the inner solar system, it must be on a very elongated orbit because one end of the orbit is close to the sun while the other end is in the distant part of the solar system. Because of this, we might expect all comets seen in the inner solar system to have extremely elliptical orbits and extremely long orbital periods that carry them again back to the Oort Cloud or the Kupier Belt. Since most of the comet's orbits are nearly in the plane of ecliptic and cross the orbits of many of the planets, short period comets frequently get close enough to a planet for the planet's gravity to change the comet's orbit about the sun in the sorts of chaotic encounters. Some long-period comets come by only once in a million years. Some might not return due to the presence of other heavenly bodies.

There is also a family of comets called as the Sun grazers, comets whose perihelia are located very close to the surface of the Sun. Sun grazers sometimes come in groups, with successive comet following in nearly identical orbits. The reason might be the destruction of a large comet.

Central force motion

When a mass particle possesses an accelerated motion and if the resultant force causing this accelerated motion always passes through a fixed point such a motion is called central force motion. The fixed point is called centre of force.

Consider a particle of the mass m, whose position vector is $\overrightarrow{r}$, relative to a fixed point O. Let the particle trace a curve C under the action of the central force $\overrightarrow{F}$, which may be directed towards or away from the origin,

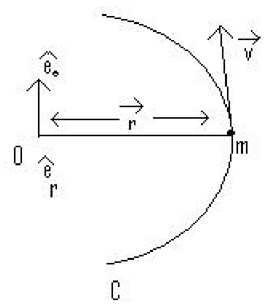

Figure 2: Central Force

we can equate as

$$F\hat{e_r} = m\dot{\overrightarrow{v}} \tag{11}$$

$$\Longrightarrow \overrightarrow{r} \times \; m\dot{\overrightarrow{v}} \tag{12}$$

from Kepler' s laws of motion, areal velocity

$$\Longrightarrow \dot{\overrightarrow{A}} = \frac{1}{2}\overrightarrow{r} \times \; \overrightarrow{v} \tag{13}$$

$$\Longrightarrow \dot{\overrightarrow{A}} = \frac{1}{2}\, r^2 \; \dot{\theta}\hat{e_\theta} = \frac{1}{2}\overrightarrow{h} \tag{14}$$

$$\Longrightarrow \dot{A} = \frac{1}{2}\, r^2 \; \dot{\theta} = \frac{1}{2}\, h \tag{15}$$

here h is the angular momentum per unitmass.

$$\Longrightarrow A = \frac{1}{2}\, h \; t + constant \tag{16}$$

This is a verification for Kepler's second law.

Integrals of Energy

If force F depends only on the length of the radius vector as

$$F = F(r) \tag{17}$$

Then we have

$$\frac{1}{2}\, m\ v^2 = \int F(r)dr\ \ + E \tag{18}$$

on considering a conservative force field, that is,

$$\overrightarrow{F} = -\nabla U \tag{19}$$

$$\implies\ F = -\frac{\partial U}{\partial r} \tag{20}$$

$$\implies\ \frac{1}{2}\, m\ v^2 + U(r) = E \tag{21}$$

This equation states that the sum of the kinetic energy and the potential energy of a moving particle under central force field is constant.

Differential Equation of the orbit

Under the central force motion, both the angular momentum and the total energy of the system are constants and we have

$$r^2\dot{\theta} = h \tag{22}$$

and

$$\frac{1}{2}\,(\dot{r}^2 + r^2\dot{\theta}^2) + U(r) = E \tag{23}$$

$$\implies\ \frac{dr}{d\theta} = \frac{\dot{r}}{\dot{\theta}} \tag{24}$$

putting

$$u = \frac{1}{r}, \frac{du}{d\theta} = -\frac{\dot{r}}{h}\ and \frac{d^2\ u}{d\ \theta^2} = -\frac{1}{h^2\ u^2}\frac{d^2\ r}{d\ t^2} \tag{25}$$

$$\implies\ m(\ddot{r} - h^2\ u^3) = F(U) = -m(h^2\ u^2\frac{d^2\ u}{d\ \theta^2} + h^2\ u^3) \tag{26}$$

$$\frac{d^2\ u}{d\ \theta^2} + U = -\frac{1}{m\ h^2\ u^2}F(u) \tag{27}$$

This is a second order differential equation in u as a function of θ and is known as the polar equation of the orbit. If a proper force is specified then the appropriate equation of the orbit can be obtained. In almost all astronomical applications, the orbits of the planets arise from the inverse square forces, as postulated by Newton.

$$F(r) = -\frac{GMm}{r^2} \, , we \ get \tag{28}$$

$$\frac{d^2 \ u}{d \ \theta^2} \ + U = -\frac{GMm}{r^2} \tag{29}$$

or

$$U = \frac{GM}{h^2} \ + Acos(\theta - \theta_0) \ , A \ and \ \theta_0 \ areconstants \tag{30}$$

$$\implies r = \frac{\dfrac{h^2}{GM}}{1 + \dfrac{A \ h^2 \ cos(\theta - \theta_0)}{GM}} \tag{31}$$

which is of the form,

$$r = \frac{P}{1 + cos(\theta - \theta_0)} \tag{32}$$

a standard equation of a conic, p is the semi-latus rectum.
where

$$p = \frac{h^2}{GM} \ , e = \frac{A \ h^2}{GM} \tag{33}$$

Here e is the eccentricity of the conic, p/e is the distance of the focus from the directrix.For

e<1, the conic is ellipse

e=1, the conic is parabola

e>1, the conic is hyperbola

On introducing the value of E in the same equations and finding the solution we end up with a solution whose eccentricity e is given as

$$e = [1 + \frac{2Eh^2}{mG^2M^2}]^{\frac{1}{2}} \tag{34}$$

ie, If E=0, e=1, the orbit is a parabola

E<0, e<1, the orbit is an ellipse

E>0, e>1, the orbit is a hyperbola

Geometry of a parabolic orbit

When E=0, e=1, the orbit is a parabola. A parabolic orbit rarely occurs in nature, although the orbits of many comets are nearly of the parabolic type. It may be noted that an object or a particle travelling along a parabolic path is on a one way trip to infinity and beyond.

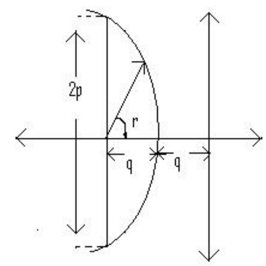

Figure 3: Parabolic Orbit

Here

$$e = 1 \ , \ q = \frac{p}{2} \tag{35}$$

Where q is the distance from the focus to vertex and p is the semi latus rectum. The equation of a parabolic orbit is given as

$$r = \frac{2q}{1 + cos(\theta - \theta_0)} = \frac{p}{1 + cos(\theta - \theta_0)} \tag{36}$$

since E=0 , we have

$$E = \frac{1}{2}mv^2 - \frac{GMm}{r} = 0 \tag{37}$$

$$\implies \frac{1}{2}v^2 = \frac{GMm}{r} \implies v = (\frac{2GM}{r})^{\frac{1}{2}} \tag{38}$$

This velocity is called escape velocity. The escape velocity is the velocity required by any object of any mass to escape from the gravitational field at a specific distance. It is independent of mass of particle .The particle moving with escape velocity or nearly escape velocity will have almost a parabolic orbit or path.

Position in parabolic orbit

The parabolic orbits are rarely found in nature. However, the orbits of some comets are nearly parabolic. An object travelling in a parabolic path is on a one way trip to infinity and will never retrace the same path again. To find the position (r,ν) of an object in a parabolic path as a function of time, we may recall that is absolute angular momentum is a constant. That is

$$r^2\frac{d\nu}{dt} = h = \sqrt{\rho\nu} \; where \; \nu = GM \tag{39}$$

The general equation of the conic is

$$\frac{p}{r} = 1 + ecos\nu \tag{40}$$

eliminating r from between both equation we get

$$\frac{d\nu}{(1 + ecos\nu)^2} = \sqrt{\frac{\mu}{p^3}}dt \tag{41}$$

Thus, in the case of parabolic orbit, when e=1 , p=2q we have

$$\frac{2q}{r} = 1 + cos\nu = 2cos^2(\frac{\nu}{2}) \tag{42}$$

or

$$r = qsec^2(\frac{\nu}{2}) \tag{43}$$

here q is the distance of the focus from the vertex and ν is the true anomaly. again we have

$$\frac{d\nu}{(1 + cos\nu)^2} = \sqrt{\frac{\mu}{p^3}}dt \tag{44}$$

it can be reduced to

$$\frac{1}{4}sec^4(\frac{\nu}{2})d\nu = \sqrt{\frac{GM}{p^3}}dt = \sqrt{\frac{GM}{2^3q^3}}dt \tag{45}$$

or

$$sec^4(\frac{\nu}{2})d\nu = \sqrt{\frac{2GM}{q^3}}dt \tag{46}$$

That is

$$[1 + tan^2(\frac{\nu}{2})]sec^2(\frac{\nu}{2})d(\frac{\nu}{2}) = \sqrt{\frac{GM}{2q^3}}dt \tag{47}$$

on integrating

$$\frac{1}{3}tan^3(\frac{\nu}{2}) + tan(\frac{\nu}{2}) = \sqrt{\frac{GM}{2q^3}}t + constant \tag{48}$$

at perihelion, at t=T we have

$$constant = -\sqrt{\frac{GM}{2q^3}}t \tag{49}$$

Thus we arrive at a cubic equation

$$\frac{1}{3}tan^3(\frac{\nu}{2}) + tan(\frac{\nu}{2}) = \sqrt{\frac{GM}{2q^3}}(t - T) \tag{50}$$

This equation is known as Baker' s equation, which gives a relation between true anomaly ν and time t for a parabolic orbit. This cubic equation can be solved numerically to get ν as a function of t. Also the radius vector

$$r = qsec^2(\frac{\nu}{2}) = q(1 + tan^2(\frac{\nu}{2})) \tag{51}$$

By putting T=0 ,ie impact or perihelion time as zero we have Baker' s equation as

$$\frac{1}{3}tan^3(\frac{\nu}{2}) + tan(\frac{\nu}{2}) = \sqrt{\frac{GM}{2q^3}}t \tag{52}$$

or time

$$t = \sqrt{\frac{2 \times q^3}{GM}}[\frac{1}{3}tan^3(\frac{\nu}{2}) + tan(\frac{\nu}{2})] \tag{53}$$

ie, we can predict the time at which the object reaches a distance r, since r=r(ν)

For real life situation we have methods to find out the distance between the object and source of the central force. For comets, sun's gravity is the source of central force and r is the distance between sun and the comet. Also by methods of spherical astronomy we can find the value of true anomaly. If a position and true anomaly is available then it is easy to find the perihelion distance as

$$q = rcos^2(\frac{\nu}{2}) \tag{54}$$

4 Comet ISON-C/2012 S1

C/2012 S1, also known as Comet ISON or Comet Nevski-Novichonok is a sungrazing comet discovered on 21 September 2012 by Vitali Nevski and Artyon Novichonok. The discovery was made using the .4 meter(16 in) reflector of the International Scientific Optical Network near Kislovodsk, Russia and the automated asteroid- discovery program CoLitec. Precovery images by the Mount Lemmon Survecy from 28 December 2011 and by Pan- stars from 28 January 2012 were quickly located. The expected perihelion distance is 0.01244AU or 1.861×10^9 m

Perihelion : 0.01244 AU Year : 2013

4.1 Distance and Velocity of C/2012 S1

Table 1: Time Distance and Velocity of C/2012 S1

Sl No.	**Date**	**Month**	**Distance from Sun (AU)**	**Distance from Earth (AU)**	**Velocity w.r.t to Sun (Km/s)**
1	1	February	4.927	4.029	19.37
2	1	March	4.616	4.037	20.01
3	1	April	4.258	4.193	20.84
4	1	May	3.897	4.324	21.78
5	1	June	3.505	4.312	22.96
6	1	July	3.105	4.097	24.39
7	1	August	2.661	3.644	26.35
8	1	September	2.177	2.971	29.14
9	1	October	1.651	2.151	33.46
10	10	October	1.471		35.45
11	10	October	1.464		35.57

Table 2: Escape Velocity of C/2012 S1

Sl No.	**Month,Date**	**Distance from Sun** ($\times 10^{11}$)**(m)**	**Escapevelocity(Km/s)**
1	Feb,1	7.37	19.03
2	Mach,1	6.91	19.65
3	April,1	6.37	20.47
4	May,1	5.83	21.39
5	June,1	5.24	22.57
6	July,1	4.64	23.98
7	August,1	3.98	25.89
8	September,1	3.25	28.66
9	October,1	2.46	32.94
10	October,10	2.2	34.83
11	October,10	2.19	34.91

Escape velocity

$$v_{escape} = \sqrt{\frac{2GM}{r}} \tag{55}$$

Since the value of real velocity available is greater than escape velocity the orbit is strictly a hyperbola. But the difference is not so large, ie., the real velocity is almost in agreement with the escape velocity. We could say that the path is parabolic. The reason for the variation might be the difference in the Sun's mass or the presence of other planets in the solar system, mainly because of Jupiter and Saturn.

Since we assumed the path is almost parabolic we could take the value of e as e=1.

Then $r = qsec^2(\nu/2)$, the values are as follows:

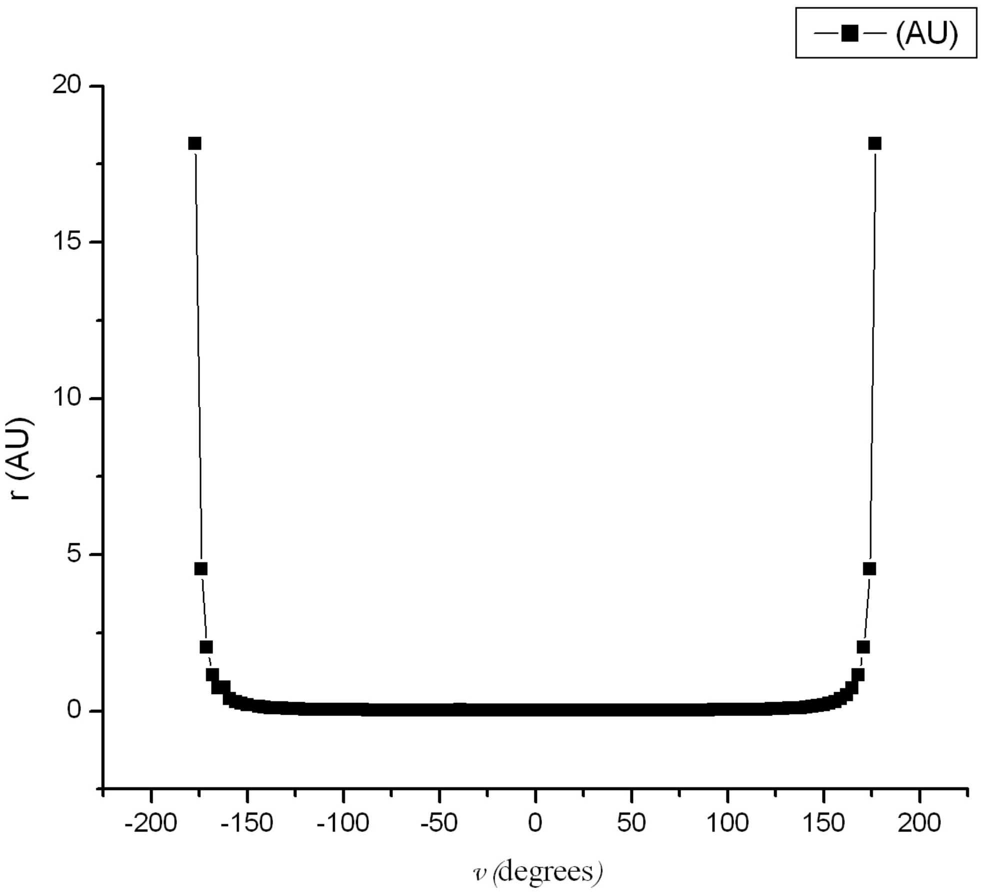

Figure 4: Possible Path ofC/2012 S1

Table 3: Escape velocity with respect to Sun (Year 2013)

Sl No.	ν (degrees)	q(0)(AU)	r(AU)	Sl No.	ν (degrees)	q(0)(AU)	r(AU)	Sl No.	ν (degrees)	q(0)(AU)	r(AU)
1	177	0.01244	18.154	42	51	0.01244	0.0153	83	-75	0.01244	0.0198
2	174	0.01244	4.542	43	48	0.01244	0.0149	84	-78	0.01244	0.0206
3	171	0.01244	2.021	44	45	0.01244	0.0146	85	-81	0.01244	0.0215
4	168	0.01244	1.138	45	42	0.01244	0.0143	86	-84	0.01244	0.0225
5	165	0.01244	0.73	46	39	0.01244	0.01399	87	-87	0.01244	0.0236
6	162	0.01244	0.508	47	36	0.01244	0.0138	88	-90	0.01244	0.0299
7	159	0.01244	0.374	48	33	0.01244	0.0135	88	-93	0.01244	0.0263
8	156	0.01244	0.288	49	30	0.01244	0.01333	90	-96	0.01244	0.0278
9	153	0.01244	0.228	50	27	0.01244	0.0132	91	-99	0.01244	0.0295
10	150	0.01244	0.186	51	24	0.01244	0.013	92	-102	0.01244	0.0314
11	147	0.01244	0.154	52	21	0.01244	0.0129	93	-105	0.01244	0.0336
12	144	0.01244	0.13	53	18	0.01244	0.0127	94	-108	0.01244	0.036
13	141	0.01244	0.112	54	15	0.01244	0.0126	95	-111	0.01244	0.0388
14	138	0.01244	0.097	55	12	0.01244	0.01257	96	-114	0.01244	0.0419
15	135	0.01244	0.085	56	9	0.01244	0.01252	97	-117	0.01244	0.0456
16	132	0.01244	0.075	57	6	0.01244	0.01247	98	-120	0.01244	0.0498
17	129	0.01244	0.067	58	3	0.01244	0.01245	99	-123	0.01244	0.055
18	126	0.01244	0.06	59	0	0.01244	0.01247	101	-126	0.01244	0.06
19	123	0.01244	0.055	60	-3	0.01244	0.01245	102	-129	0.01244	0.067
20	120	0.01244	0.0498	61	-6	0.01244	0.01247	103	-132	0.01244	0.075
21	117	0.01244	0.0456	62	-9	0.01244	0.01252	104	-135	0.01244	0.085
22	114	0.01244	0.0419	63	-12	0.01244	0.01257	105	-138	0.01244	0.097
23	111	0.01244	0.0388	64	-15	0.01244	0.0126	106	-141	0.01244	0.112
24	108	0.01244	0.036	65	-18	0.01244	0.0127	107	-144	0.01244	0.13
25	105	0.01244	0.0336	66	-21	0.01244	0.0129	108	-147	0.01244	0.154
26	102	0.01244	0.0314	67	-24	0.01244	0.013	109	-150	0.01244	0.186
27	99	0.01244	0.0295	68	-27	0.01244	0.0132	110	-153	0.01244	0.228
28	96	0.01244	0.0278	69	-30	0.01244	0.0133	111	-156	0.01244	0.288
29	93	0.01244	0.0263	70	-33	0.01244	0.0135	112	-159	0.01244	0.374
30	90	0.01244	0.0249	71	-36	0.01244	0.0128	113	-162	0.01244	0.7508
31	87	0.01244	0.0236	72	-39	0.01244	0.03999	114	-165	0.01244	0.73
32	84	0.01244	0.0225	73	-42	0.01244	0.0143	115	-168	0.01244	1.138
33	81	0.01244	0.0215	74	-45	0.01244	0.0146	116	-171	0.01244	2.021
34	78	0.01244	0.0206	75	-48	0.01244	0.0149	117	-174	0.01244	4.542
35	75	0.01244	0.0198	76	-51	0.01244	0.0153	118	-177	0.01244	18.154
36	72	0.01244	0.019	77	-54	0.01244	0.0157				
37	69	0.01244	0.0183	78	-57	0.01244	0.0161				
38	66	0.01244	0.0177	79	-60	0.01244	0.0166				
39	63	0.01244	0.0171	80	-63	0.01244	0.0171				
40	60	0.01244	0.0166	81	-66	0.01244	0.0177				
41	57	0.01244	0.0161	82	-69	0.01244	0.0183				

Angular momentum, $h = \nu r sin\phi$ Where ϕ is the angle made by the radius vector and velocity component (vector), at perihelion r=0.01244 AU And since the path is almost parabolic the approximate value of velocity is given

$$\text{as } v_{esc} = \sqrt{(\frac{2GM}{0.01244AU})}$$

$$v_{esc} = 378.7\ km/s$$

$$v_{esc} \approx 380\ km/s$$

Therefore Angular momentum's approximate value for unit mass

$$h = v_{esc} \times 0.01244AU$$

here $sin\phi = 1$, since at perihelion $\phi = \pi/2$ or 90^0

$$h = 7.07 \times 10^{14}$$

by knowing ϕ and r along with h we could calculate the value of object' s velocity

Now we need to predict the time of perihelion.

We already derived the Baker' s formula as

$$\frac{1}{3}tan^3(\frac{\nu}{2}) + tan(\frac{\nu}{2}) = \sqrt{\frac{GM}{2q^3}}t$$

Here we took the impact or perihelion time as zero

We have G=$6.673\times10^{-11}\ Nm^2/kg^2$

$$M = 2 \times 10^30\ kg$$

$q = 1.801 \times 10^9 m$

Now let us take a known distance for a known day Say February, 1, 2013 that is r=4.927 AU

Therefore corresponding $\nu = 174.239^0$

$$t = \sqrt{\frac{2 \times q^3}{GM}}[\frac{1}{3}tan^3(\frac{\nu}{2}) + tan(\frac{\nu}{2})] = 9827.86(2637.89)$$

$$t = 25924813.62\ seconds$$

$$t = 25924813.62/(23.93 \times 3600)\ days$$

$t = 300.93\ days$

From February 1, 2013 the $300^{th} day$ is November 27^{th} since it is 300.93 days we could take it as $28^{th} November$. That is, Perihelion will occur on 28^{th} November 2013.

Table 4: True anomaly and Distance from Sun of C/2012 S1

Sl No.	**True anomaly**	**Distance from Sun(m)**	**No. of days, TI, FI**	**Calender Day**
1	± 180	∞		
2	160	0.412	TI-8	November 20
3	140	0.106	TI-2	November 26
4	120	0.0498	TI-0.395	November 28
5	100	0.0301	TI-O.200	November 28
6	80	0.0212	TI-0.118	November 28
7	60	0.0166	TI-0.073	November 28
8	40	0.0141	TI-0.043	November 28
9	20	0.0128	TI-0.020	November 28
10	0	0.01244	I -0	November 28
11	-20	0.0128	FI-0.020	November 28
12	-40	0.0141	FI-0.043	November 28
13	-60	0.0166	FI-0.073	November 28
14	-80	0.0212	FI-0.118	November 28
15	-100	0.0301	FI-0.200	November 28
16	-120	0.0498	FI-0.395	November 28
17	-140	0.106	FI-2	November 30
18	-160	0.412	FI-8	December 6

Table 5: Ture anomaly and Calender Date

Sl No.	Distance from Sun	True anomaly	No. of days	Calender date	Year	v_esc Km/s
1	2.0	170.953	78.236	September 11	2013	
2	1.8	170.463	66.877	September 23	2013	
3	1.6	169.883	56.117	October 3	2013	
4	1.4	169.182	45.9999	October 14	2013	
5	1.2	168.312	36.579	October 23	2013	
6	1.0	167.192	27.911	November 1	2013	
7	0.8	165.673	20.062	November 8	2013	
8	0.6	163.442	13.127	November 15	2013	
9	0.4	159.685	7.249	November 21	2013	
10	0.2	151.116	2.669	November 26	2013	
11	0.01244	0	0	November 28	2013	378.7
12	0.2	151.116	2.669	December 1	2013	94.45
13	0.4	159.685	7.249	December 6	2013	66.79
14	0.6	163.442	13.127	December12	2013	54.53
15	0.8	165.673	20.062	December 19	2013	47.22
16	1.0	167.192	27.911	December 26	2013	42.24
17	1.2	168.312	36.579	January 3	2014	38.56
18	1.4	169.182	45.9999	January 13	2014	35.7
19	1.6	169.883	56.117	January 24	2014	33.39
20	1.8	170.463	66.870	February 3	2014	31.48
21	2.0	170.953	78.236	February 15	2014	28.87
22	2.2	171.375	90.189	February 27	2014	28.48
23	2.4	171.743	102.698	March 11	2014	27.26
24	2.6	172.067	115.711	March 24	2014	26.19
25	2.8	172.356	129.249	April 7	2014	25.24
26	3	172.616	143.305	April 21	2014	24.39
27	3.2	172.850	157.759	May 5	2014	23.61
28	3.4	173.064	172.378	May 20	2014	22.91
29	3.6	173.2599	188.174	June 5	2014	22.26
30	3.8	173.4399	204.019	June 21	2014	21.67
31	4	173.606	220.255	July 7	2014	21.12
32	4.2	173.760	236.898	July 23	2014	20.61
33	4.4	173.904	254.017	August 10	2014	20.14
34	4.6	174.038	271.467	August 27	2014	19.69

4.2 Variation in Distance and Velocity

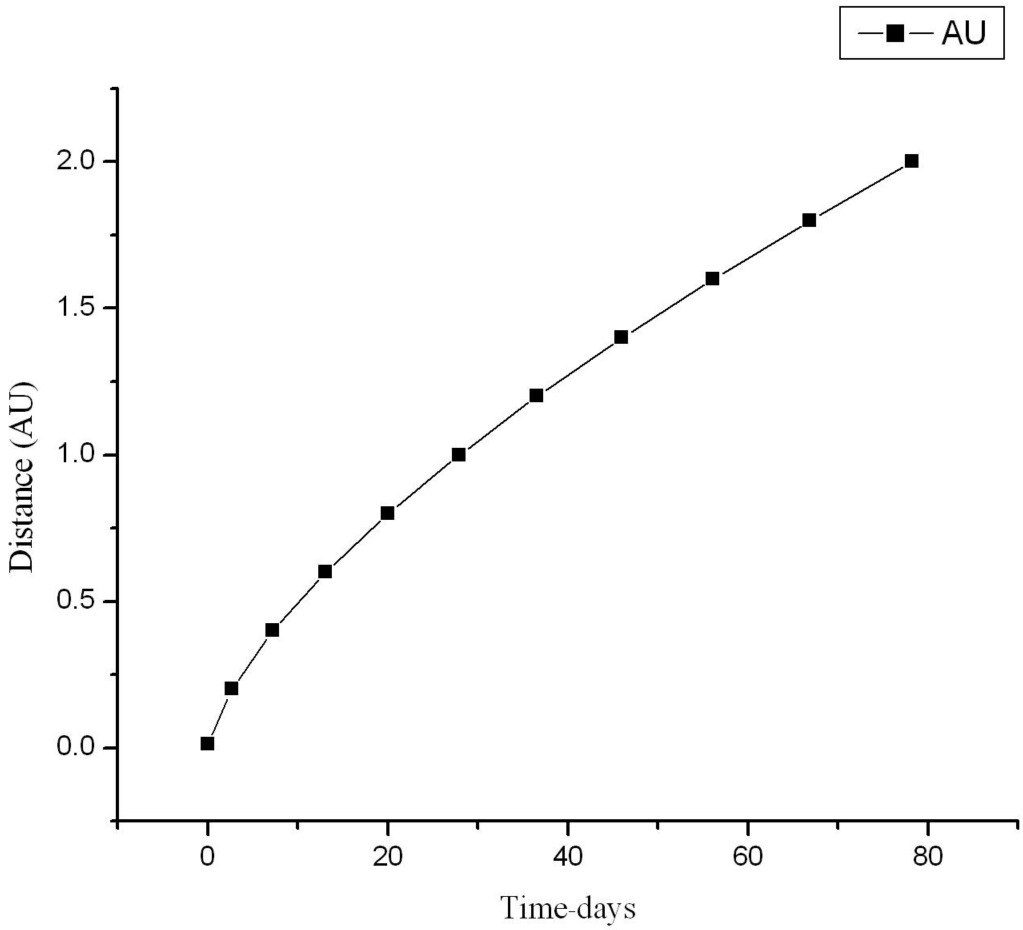

Figure 5: Distance-Time graph after perihelion

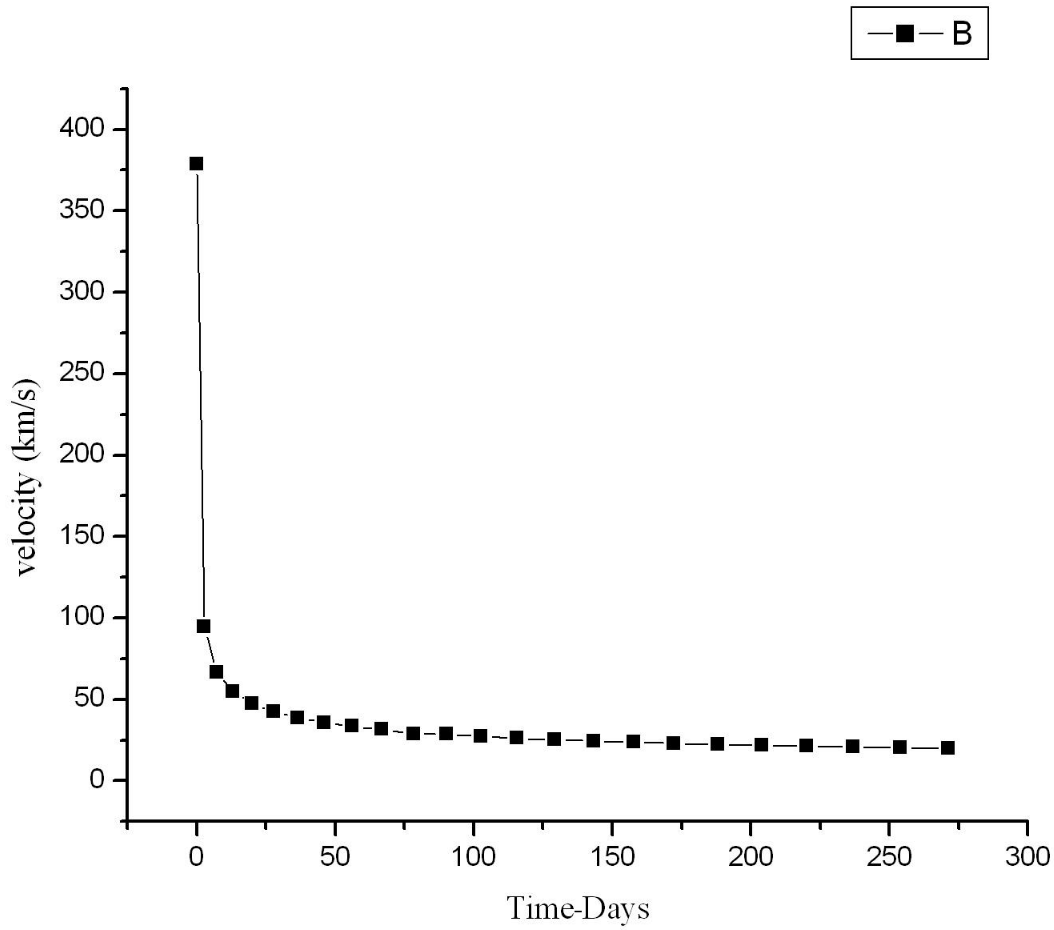

Figure 6: Velocity-Time graph after perihelion

5 Comet ISON Results

Comet ISON also known as Comet Nevski- Novichonok is a sungrazing comet discovered on 21 September 2012 by Vitali Nevski and Artyon Novichonok. The discovery was announced by the Minos Planet Centre on 24^{th} September. Observations by SWIFT suggest that C/2012 S1's nucleus is around 5 kilometers in diameter.

The comet has almost a parabolic orbit, accurately it is hyperbolic. C/2012 S1 will come to perihelion on 28^{th} November 2013 at a distance of 0.01244 AU. On that day the comet is expected to go around an angle of 240^0 around sun, with a velocity approximately 380 km/s. By August 27, 2014 the comet will be at a distance approximately 4.6 AU from Sun. Since the orbit is nearly parabolic the comet shall never come back again. Also the inclination of 62.39^0 from solar plane, the comet might be from the Oort cloud. At perihelion, the temperature may reach high enough to melt iron. Also, it will come within the Roche limit, meaning it may disintegrate due to Sun's gravity. The fate might be similar to that of Shoemaker-Levy 9.

The obtained results may vary due to the crude approximation. The mass of Sun is one such approximation taken. Even the original velocity is greater than escape velocity, but almost equal, escape velocity is taken for calculation. The eccentricity is not strictly 1 but approximately 1. The presence of the other bodies in the solar system is not considered since the total mass of the solar system almost equals the Sun's. Considering all these facts we could get much more exact results. Even though the results are fairly good enough for a rough calculation.

"From the theoretical point of view one would think that monopoles should exist, because of the prettiness of the mathematics. Many attempts to find them have been made, all of them unsuccessful. One should conclude that pretty mathematics by itself is not an adequate reason for nature to have made use of a theory"

Paul .A.M.Dirac

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