Definition of the Derivative

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The derivative is a useful definiton because it gives the tangent line to the given equation. The derivative is a basic term that is applied in most advanced mathematical concepts such as Abstract Algebra, Differential Equations, Electrodynamics, and other advance courses.

Definition 1 (The Formal Definition of Derivative) *The derivative is defined as the computation of the slope of a tangent line, the instantaneous rate of change of a continuous function, and the instantaneous velocity of an object. The derivative of $f(x)$ with respect to x is the function f' and is defined as,*

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \tag{1}$$

Definition 2 (The Informal Definition of Derivative) *The derivative of a continuous function is the rate of change of a function at a given input. The derivative of a function can also be denoted as*

$$(f^n(x))' = n \cdot f^{n-1}(x) \tag{2}$$

Example 1 (Solution Attainable) *Find the derivative of the function using the limit definition:*

$$f(x) = x^2 + 7 \tag{3}$$

Solution:

$$\begin{aligned}
f'(x) &= \lim_{h \to 0} \frac{((x+h)^2 + 7) - (x^2 + 7)}{h} \\
&= \lim_{h \to 0} \frac{x^2 + 2xh + h^2 + 7 - x^2 - 7)}{h} \\
&= \lim_{h \to 0} \frac{2xh + h^2}{h} \\
&= \lim_{h \to 0} \frac{h(2x + h)}{h} \\
&= \lim_{h \to 0} 2x + h \\
&= 2x
\end{aligned}$$

Example 2 (Solution Unattainable) *Find the derivative of the function using the limit definition:*

$$f(x) = \begin{cases} x^2 & x > 0 \\ -x^3 & x < 0 \end{cases} \tag{4}$$

Solution: Since the function above is a piece wise function, there is a point at x = 0 where the function has a hole. Since the function is discontinues, the derivative does not exist.