Entropy Minimization Based Synchronization Algorithm for Underwater Acoustic Receivers

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ABSTRACT

This paper presents a new entropy minimization criterion and corresponding algorithms that are used for both symbol timing and carrier frequency recovery for underwater acoustic receivers. It relies on the entropy estimation of the eye diagram and the constellation diagram of the received signal. During the parameter search, when perfect synchronization is achieved, the entropy will reach a global minimum, indicating the least intersymbol interference or a restored constellation diagram. Unlike other synchronization methods, this unified criterion can be used to build an all-in-one synchronizer with high accuracy. The feasibility of this method is proven using a theoretical analysis and supported by sea trial measurement data.

1. INTRODUCTION

In a digital receiver for underwater acoustic communication, the down-sampler is located after the matched filter. It is triggered by a local clock to sample the filter output at multiples of the symbol period. In the ideal case, the local clock is synchronized with the real symbol clock, however, since the symbol clock is usually unknown to the receiver, a symbol timing recovery circuit is required. Particularly in the underwater acoustic channel, the symbol clock period is time variant. In addition, the relative motion will shift the carrier frequency at passband, a phenomenon known as the Doppler shift. Therefore a continuous estimation and compensation of the synchronization error is essential to maintain the link reliability.

The symbol clock has two fundamental parameters: symbol rate and symbol timing phase. These parameters can be estimated either in feedback or feedforward configurations [1]. The feedback configuration features a timing error detector, that produces a signal proportional to the difference between the local timing phase and the actual symbol clock phase. The filtered error signal is fed back to adjust the down-sampling until the detector output is minimized. On the other hand, the feedforward configuration estimates an optimal synchronization offset, which is directly used for down-conversion. As such, this technique allows fast acquisition, since it does not rely on a control loop with slow time constants. This paper is focused on the feedforward configuration to track rapidly changing propagation conditions.

Feedforward schemes recover the symbol clock by finding the periodicity of the received signal time series. To extract the frequency and phase of a periodic signal, Fourier analysis is a standard technique. However, it is not suitable for bandwidth efficient communication systems, particularly because the received signal is not periodic. In these conditions, the square timing recovery method [2] is probably the most popular solution. Indeed, since a linearly modulated waveform is second-order cyclostationary [3], the magnitude square can be considered as a second-order nonlinear transform. As such, the square operation converts a cyclostationary signal into a periodic signal, and the Fourier transform is applied on this signal for symbol timing recovery by finding the spectrum line at the symbol rate.

Since Shannon's work [4], entropy is used as a major tool in information theory. But except for theoretical analysis, it is rarely used in signal processing because of its complexity [5]. Nonetheless, it has been shown in [6] that the carrier frequency offset of the M-ary PSK signals can be recovered by minimizing the entropy of the instantaneous phase probability density function. Also, it has been reported that the error entropy minimization algorithms can be applied for channel equalization [7]. However, no application can be found in the literature for symbol timing recovery.

In this paper, we propose a new synchronization criterion making use of the periodicity of the signal entropy. The first advantage is that it can be applied to both symbol timing and carrier frequency recovery. Moreover, it only requires certain probability functions obtained from the eye diagram and constellation diagram, so the detectors described in this paper can also be used to track the clock during data detection. In addition, entropy estimation measures intersymbol interference (ISI), and includes higher-order statistics of the data [7], whereas other maximum likelihood based synchronization techniques only use second-order statistics and assume there is no ISI in the received signal [8]. This is the underlying cause of its superior performance in distorted channels.

This paper is organized as follows. In Section 2, the basics of information entropy are introduced. The entropy minimization criterion and corresponding algorithm is discussed for symbol timing recovery and it is applied to the complex signal for carrier frequency recovery. Section 3 validates the

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feasibility of the entropy minimization based synchronization algorithm with sea trial data. The conclusion is presented in Section 4.

2. ENTROPY MINIMIZATION BASED SYNCHRONIZATION ALGORITHM

In the beginning of this section the information entropy, the system parameters and terminology are briefly introduced. By analyzing the entropy of the eyediagram and constellation diagram, algorithm that can recovery both symbol timing and carrier offset are detailed.

2.1 Entropy and System parameters

The concept of entropy developed by Shannon is a measure of the amount of information or, in other words, its randomness. The simplest and most common approach uses histogram-based estimation. Assuming k possible observations of a one-dimensional discrete signal, with p_i representing the probability of the i-th possibility, the information entropy H can be expressed as

$$H = -\sum_{i=1}^{k} p_i \log_2 p_i. \tag{1}$$

Since the base of the logarithmic function is 2, the entropy is said to be in *bits*. The choice of k should be balanced between accuracy and complexity.

Before describing entropy based synchronization techniques, system parameters and terminology are defined here. In this work, it is assumed that the signal is linearly modulated (not limited to phase-shift keying) with an alphabet size of M. The bit stream follows an independent identical distribution. The pulse shaping filter is designed under the Nyquist criterion, such that there is no ISI at the proper sampling instances. The span of the shaping filter is limited to L symbols, where L is an even number. In this section, the channel distortion is neglected, the symbol rate is known and the signal is analyzed at baseband without carrier frequency offset. Some of these system parameters will be reviewed in the next few sections.

2.2 Eye Diagram Entropy and Symbol Timing Recovery

The eye diagram is a graphical illustration that consists of many overlaid traces of small sections of a signal. Normally, the length of the window is one or two symbol periods. The ideal down-sampling instant is located in the middle of the eye diagram where the eye opening reaches its maximum, indicating the optimum error immunity. Fig. 1 is an eye diagram for a BPSK signal overlaid at each symbol period. The binary data is shaped using raised-cosine pulses with a roll-off factor of 0.5.

Since the data is equally distributed in M possible symbols, the probability of the ith symbol is $p_i = 1/M$ and k is equal to M. Substituting the probability into (1), the entropy in the middle of the eye is given by

$$H_{mid} = -\sum_{i=1}^{M} \frac{1}{M} \log_2 \frac{1}{M} = \log_2 M. \tag{2}$$

It is exactly the same as the amount of information carried by each symbol. According to the Nyquist criterion, there is no ISI in the middle of the eye diagram, so the alphabet size M is the only unknown parameter in (2). However, there are L adjacent pulses that interfere with each other in its neighbouring area, so the maximum information entropy is up to

$$H_{neb} = L \log_2 M. \tag{3}$$

Since only the proper sampling instances are ISI free, one can conclude that the entropy of an eye diagram comes to a global minimum in the middle of the eye.

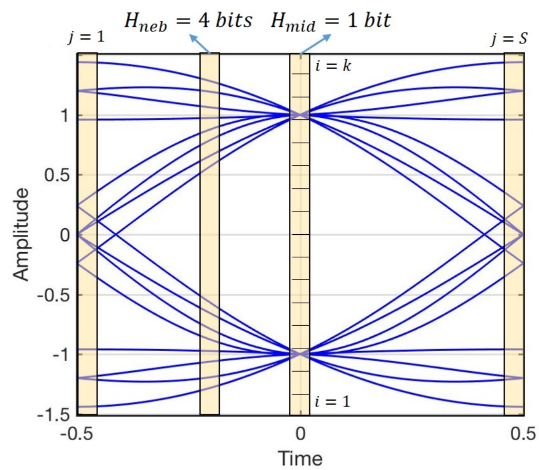


Figure 1: Eye diagrams of a BPSK signal and its timing phase entropy.

For example, Fig. 1 is an eye diagram of a BPSK signal ($M = 2$), where $L = 4$. In the middle of the eye, i.e., at instant $t = 0$, the entropy $H_{mid} = \log_2 2 = 1$ bit, while its neighbouring entropy $H_{neb} = 4 \log_2 2 = 4$ bits. In other words, the entropy of the optimum symbol timing phase is significantly lower than in its neighbouring area.

Next, let's consider some practical issues. For low symbol rate communication, such as underwater acoustic applications, it is reasonable to assume that the sample rate of the receiver's analog to digital converter (ADC) is much higher than the symbol rate, so that the down sampling can be realized using decimation instead of interpolating.

If a set of samples under observation consists of N symbols and the oversampling rate is S samples per symbol, the eye diagram data can be constructed by reshaping the samples into an N by S matrix. The one-dimensional *timing phase entropy* is found with the N samples in column j (the yellow column in Fig. 1), where $1 \leq j \leq S$. The peak-to-peak amplitude of the samples is partitioned into k bins and the entropy at the timing phase index j is given by

$$H_j = -\sum_{i=1}^{k} \frac{l_{i,j}}{N} \log_2 \frac{l_{i,j}}{N}, \tag{4}$$

where $l_{i,j}$ is defined as the number of samples falling in the i-th ($1 \leq i \leq k$) partition at phase index j.

The upper limit of number of partitions k is decided by the ADC's quantization resolution. The values are usually in binary form, so the resolution is usually expressed in bits too. Therefore, the number of output discrete values available (partitions) is expected to be a power of two. For example, an ADC with a resolution of 8 bits can encode an analog input to 256 different partitions, since $2^8 = 256$.

Timing phase estimation is achieved by searching the instance with minimum timing phase entropy in the eye diagram, while the symbol rate search needs to minimize the entropy of the whole eye diagram. The *eye diagram entropy* is given by integrating the timing phase entropy of all the S columns in the eye diagram. The eye diagram entropy is written as

$$H_{eye} = \sum_{j=1}^{S} H_j. \tag{5}$$

In (5), the timing phase entropy H_j is integrated through the eye diagram. Thus H_{eye} can be considered as a two-dimensional entropy.

For example, even if there is 1% symbol rate error, the eye diagram in Fig. 1 will be completely closed. The solution for the eye diagram entropy with symbol rate error is hard to solve analytically, but it is obvious that there is much more ISI or randomness throughout all the timing phases than that in Fig. 1. Therefore, only when the symbol rate is restored does the eye diagram become open, and correspondingly the entropy reaches its minimum.

In the entropy calculation for both symbol timing phase and symbol rate search, the algorithm requires N symbols to generate one entropy result. Since the entropy is a function of probability, to obtain relative accurate probabilities, N cannot be set to a small value. In the simulation, the results tend to be stable when N is greater than a few hundreds symbols. Below that number, the probability function may have significant bias.

2.3 Entropy of Complex Signal

When converted to baseband, the signals from in-phase and quadrature channel can generate two independent eye diagrams. However, if we consider the I/Q components as one complex signal, the two eye diagrams can be combined to generate a new Cartesian plot. For each symbol timing phase, instead of a set of scalars representing the signal amplitude, it is a set of complex data containing both the in-phase data on the real axis and the quadrature data on the imaginary axis. In fact, a diagram of such a set of complex data can be represented using a constellation diagram.

The *constellation diagram entropy* is an extension of the timing phase entropy discussed in Section 2.2. To estimate its value, the constellation diagram is partitioned into a matrix of k by k bins. Fig. 2 is an example showing the partition of a constellation diagram with perfect symbol timing recovery but small carrier frequency offset Δf. In this figure, $k = 16$ and the coordinate of the yellow bin is (7, 15). For N symbols under observation, if there are l_{xy} samples falling into bin (x, y), the constellation diagram entropy is given by

$$H_{\Delta f} = -\sum_{x=1}^{k}\sum_{y=1}^{k} \frac{l_{xy}}{N} \log_2 \frac{l_{xy}}{N}. \tag{6}$$

2.4 Frequency Offset Search

The most notable synchronization issue that is unique in passband communication is the carrier frequency recovery. It originates because of the the frequency difference between the carrier of the received signal and the local down converter. It has been mentioned in Section 1 that the entropy of the instantaneous phase probability density function can be used for carrier frequency recovery. In fact, the carrier frequency can also be restored by minimizing the constellation diagram entropy. Unlike the symbol timing recovery, the extra entropy when there is frequency offset is mainly from the rotating constellation. That is to say, by applying the entropy minimization criterion, both symbol timing and carrier frequency can be recovered. Therefore, an all-in-one synchronizer can be designed based on one single criterion.

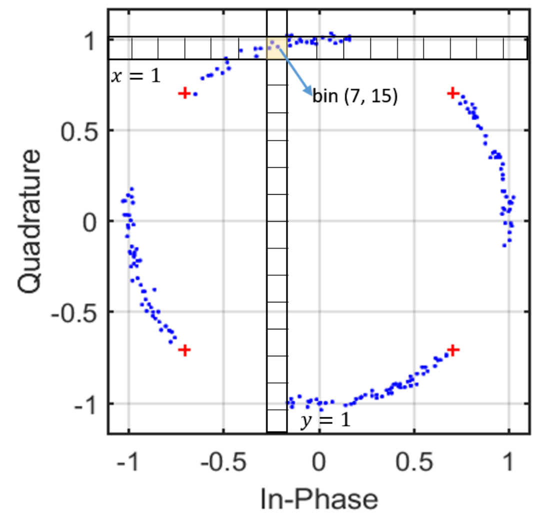


Figure 2: Constellation diagram of a QPSK signal with carrier frequency offset and its partitions for entropy estimation.

3. ALGORITHM VALIDATION WITH SEA TRIAL DATA

In this section, the entropy minimization based synchronization algorithm will be validated with sea trial measurement data.

3.1 Sea Trial Environment and settings

The sea trial took place in Northwest Arm near Halifax, NS, Canada on July 6th 2017. The experiment provided an opportunity to evaluate a variety of underwater communications algorithm. In the test area, the depth is reported to be approximately 10-13 m, and the receiver's deployment location is around 220 m away from the transmitter, and there is certain drift caused by the current. The sea state is favourable to the test. The acoustic propagation properties can also be observed using the channel impulse response in Fig. 3, where there is a strong and stable direct path of arrival and very few multipath interference.

A transmission frame is divided into pilot and data blocks. The pilot block is a wideband, linear frequency modulated chirp signal, which is used for frame synchronization. The data block is divided into four sections, and each section has a symbol rate of 240, 80, 15 and 5 baud respectively. The data consists of a pseudo-random noise (PN) sequence that has a length of 512 chips, and that is repeated 16 times. The modulation scheme used is QPSK, and the transmitted waveform is pulse shaped using a raised-cosine filter with a rolloff factor 0.5. The centre frequency is 2.048 kHz.

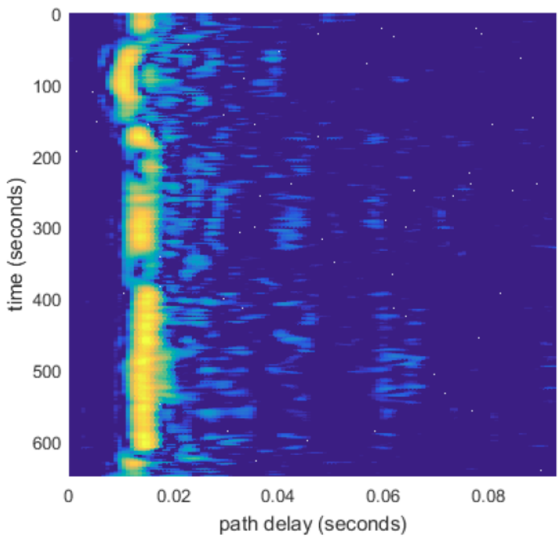
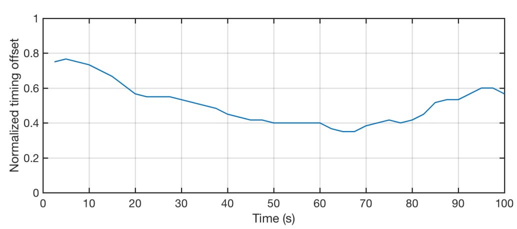


Figure 3: Channel impulse response as a function of time.

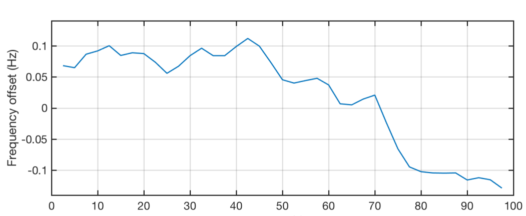
3.2 Data Processing Results

At the receiver, all signals are sampled at 44.1 kHz. The frame is first coarse synchronized with the pilot block so that the data block can be extracted. Down conversion and a matched filter are applied to the data block to generate the baseband signal. If the baseband signal is directly downsampled and demodulated, the bit error rate (BER) is 49%, which means no information is recovered.

To recover the data, the synchronization algorithm discussed in Section 2 is used before demodulation. For demonstration purposes, results from 80 baud data are presented here. In each iteration, 200 symbols are fed into the synchronizer to estimate the timing and carrier frequency offsets. The timing and frequency recovery detector outputs are shown in Fig. 4.



(a) Symbol timing offset as a function of time



(b) Carrier frequency offset as a function of time

Figure 4: Synchronization results based on entropy minimization criterion.

Several observations can be made from Fig. 4. There is a total 8192 QPSK symbols for a total duration of 102.4 s. Both symbol timing and carrier frequency offset are varying with time. One possible cause is the Doppler effect due to the relative motion between the transmitter and the receiver.

The synchronized data are depicted in Fig. 5. After demodulation, the BER is 0.41%. As a comparison, with conventional synchronization methods (pilot and PLL based), the BER is 0.49%. This result proved that the entropy minimization based synchronization algorithm is feasible and can reduce the BER.

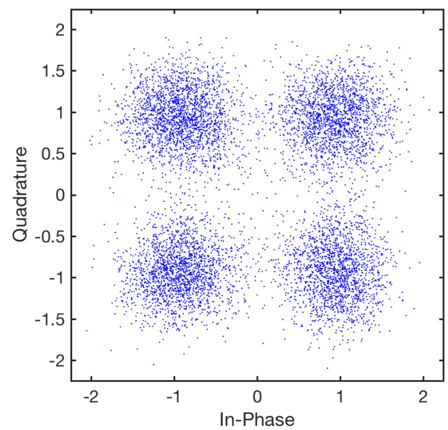


Figure 5: Scatter plot after timing and carrier recovery.

4. CONCLUSION REMARKS

A new unified synchronization criteria and algorithm for underwater acoustic coherent communication have been investigated in this paper. The entropy of the received signal's eye diagram and constellation diagram is utilized to recover both symbol timing and carrier frequency offset. In contrast to the widely used maximum likelihood based synchronization techniques, it is expected to achieve better performance due to the use of higher-order statistics. During the parameter search, when the perfect synchronization is achieved, the entropy will reach a sharp global minimum, indicating an open eye diagram or restored constellation. Therefore, an all-in-one synchronizer can be designed based on this one criterion alone. The feasibility of the proposed algorithm is tested with sea trial measurement data. The results show that they can successfully track and recover the time varying symbol timing and carrier frequency offset.

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