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Investigating the Motion of a High Speed Projectile Moving Through a Liquid

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The report includes **14** pages of the main body and **3** pages of appendices.

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Abstract

This investigation explores the conditions under which an object, moving through a liquid in high speed, has the greatest deceleration. The conditions focused on are the mass of the object, its initial velocity and its entry angle into the liquid. This is achieved through shooting pellets of varied masses and energies out of two different soft guns at a water container, recording them with a high speed camera and mapping their motion with a tracking program. The results of the experiment indicate that the lesser the mass of the object, the greater its deceleration in the fluid. The initial velocity and the entry angle of the object do not influence the rate of its speed loss. It is further explored that as the density of the fluid becomes greater, so does the deceleration of the object. The investigation is intended to deepen the understanding of the high speed motion through a medium.

Contents

1 Introduction

When a body moves through a fluid, it loses energy due to drag forces acting upon it. In this experiment, I wish to relate the loss of the speed of a body to different properties of the body, its motion and the fluid it moves through. This is done by shooting a projectile into a liquid and recording its motion with a high speed camera. The recording is then tracked and the resulting data provide the information about the rate of the speed loss in the different conditions given.

2 Method

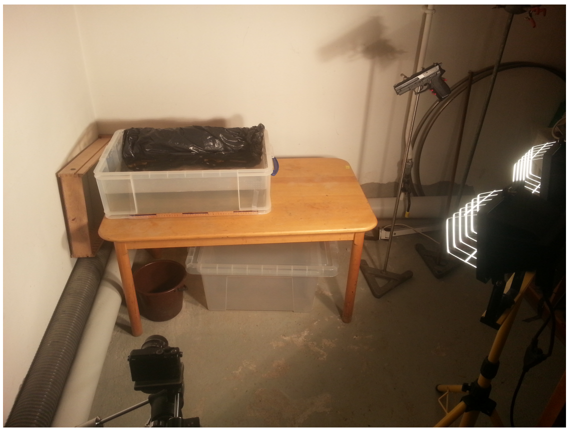


Figure 1: The setup of the experiment.

In order to measure the speed loss of the projectile, I use a high speed camera to record its motion. That recording is then processed through the program Tracker [1], in which I determine the position of the pellet for each frame and the program generates scaled values for x and y positions of the pellets, as well as the velocities v of the pellets. Performing this experiment under different conditions for the liquid (different types of liquid, such as water or soap) and for the pellet (different masses and initial velocities, different entry angles) will make it possible to investigate which conditions cause the greatest deceleration.

List of Materials:

- Spring powered soft gun, velocities given: 90m/s (for a 0.12g pellet), 70m/s (for a 0.20g pellet);
- Gas powered soft gun, velocities given: 118m/s to 135m/s;
- Gas powered soft rifle, no velocities given;
- Gas powered soft rifle, maximum velocity given: 366 m/s;
- Pellets of different masses (white: 0.256g, green: 0.365g, black: 0.419g for the soft guns and 0.494g, 0.682g and 0.991g for the rifles);
- A transparent plastic container;
- Water;
- A high speed camera (1200 fps).

The masses of the pellets have been weighed by a digital scale by weighing 10 of them and assigning the average weight to each of them:

$$m_{avg} = \frac{m_1 + m_2 + ... + m_n}{n}$$

where $n = 10$ and m_{avg} is the average mass of one pellet. The uncertainty on the mass is given by ± 0.001g, as it was measured with a digital scale precise to 0.001g.

2.1 Finding the Energies of the Soft Guns and Rifles

I made an attempt to find the initial velocity of the soft guns and rifles by making a recording of a pellet shot through air from each of them, and then finding the velocity of the pellet with the help of the Tracker program. However, once the recordings were taken, it turned out that there are gaps in the data every five frames (see Figure 2). I observed this trend in all of the videos taken and it can also be shown on a position of the pellet over time graph: in Figure 3, it can be clearly seen that the two test runs seem to have similar velocity, however the gap that occurs every fifth data point skews the slope of the possible linear fit through the data, and so the resulting velocity of the pellet.

Figure 2: Nine frames from one of the recordings of the spring powered soft gun shooting a pellet through air that were layered on top of each other.

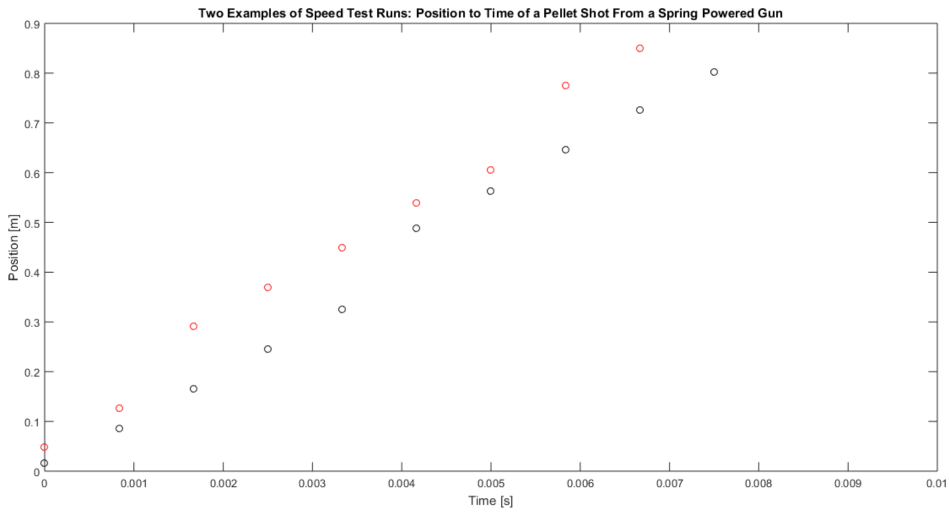


Figure 3: Graph representing the position of the pellet shot out of the spring powered gun through air over time. Data from the experiments number 1 and 5 are showed in red and black respectively.

In order to avoid a large error on the initial velocities of the soft guns, they had to be determined using a different method. Therefore, I decided to determine the velocities of the hand guns with the help of time gates and the velocities of the rifles with a large ballistic pendulum.

For the hand guns, two time gates have been placed in a line with a distance of 0.20 ± 0.01m between them. Five 0.256g pellets have been shot through the gates and the time that took them to travel in between them has been recorded (see Figure 4). Since there were only two time gates, the calculated velocity was the average velocity of the pellet within that distance, not the instantaneous one. However, it can be argued from Figures 2 and 3 that, excluding the error caused by the camera recording, the velocity of the pellet stays constant over at least the first meter of its flight. Thus the velocities measured with this method will not be significantly different from the instantaneous velocities of the hand guns.

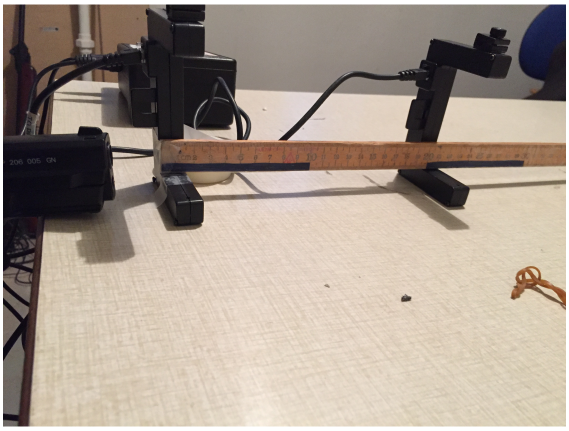


Figure 4: The setup of the experiment which was to determine the speed of a pellet shot from a hand soft gun through air.

For the rifles, during the test runs I have noted that they are too powerful for the high speed camera to record the motion of the pellets shot out of them. Since the time gates could not register the pellets either, I used a large ballistic pendulum of length $l = 1.00 \pm 0.01$m and mass $M = 206.58 \pm 0.01$g. With the high speed camera, I have taken a recording of the pellet being shot from the two rifles at the pendulum. This way, I could find the change in height of the pendulum and I could use the conservation of mechanical energy principle in finding the initial velocity of the pellet (detailed derivation in Appendix B). The setup of the pendulum is shown in Appendix A.

In order to find the velocity of the pellet shot out of the spring powered gun, I used the formula $v = \frac{\Delta x}{\Delta t}$ (since the pellet was shot horizontally, only the x displacement is included), where t is time and x is the distance between the gates. I found the time uncertainty by taking the average of t_{max} and t_{min}. In error analysis, the propagation of error for some $R = R(X, Y, ...)$ is given by[2]:

$$\delta R = \sqrt{\left(\frac{\partial R}{\partial X} \cdot \delta X\right)^2 + \left(\frac{\partial R}{\partial Y} \cdot \delta Y\right)^2 + ...}$$

Therefore I determined the uncertainty on the velocity by the propagation of error as follows:

$$\delta v = \sqrt{\left(\frac{\partial v}{\partial x} \cdot \delta x\right)^2 + \left(\frac{\partial v}{\partial t} \cdot \delta t\right)^2} = \sqrt{(t \cdot \delta x)^2 + \left(\left(-\frac{x}{t^2}\right) \cdot \delta t\right)^2}$$

In order to find the velocity of the pellet shot out of the rifles, the following formula was used:

$$v = \sqrt{2 \cdot g \cdot \Delta h} \cdot \frac{M + m}{m}$$

where g is the acceleration due to gravity, Δh is the maximum y-displacement of the pendulum from the equilibrium position, M is the mass of the pendulum and m is the mass of the pellet (derived in Appendix B). Since g is a well known quantity, the error on the measurement on it is negligible. Therefore the uncertainty on v can be found by:

$$\delta v = \sqrt{\left(\frac{\partial h}{\partial v} \cdot \delta h\right)^2 + \left(\frac{\partial m}{\partial v} \cdot \delta m\right)^2 + \left(\frac{\partial M}{\partial v} \cdot \delta M\right)^2}$$

$$\delta v = \sqrt{\left(-\sqrt{\frac{g}{2h}} \cdot \frac{M + m}{m} \cdot \delta h\right)^2 + \left(-\frac{\sqrt{2gh} \cdot M}{m^2} \cdot \delta m\right)^2 + \left(\frac{\sqrt{2gh}}{m} \cdot \delta M\right)^2}$$

The kinetic energies of the pellets shot from the different weapons can be determined by the formula

$$K = \frac{1}{2} \cdot m \cdot v^2$$

where m is the mass of the pellet. The uncertainty on the kinetic energy can be again found from the propagation of error:

$$\delta K = \sqrt{\left(\frac{\partial K}{\partial v} \cdot \delta v\right)^2 + \left(\frac{\partial K}{\partial m} \cdot \delta m\right)^2} = \sqrt{\left(m \cdot v \cdot \delta v\right)^2 + \left(\frac{v^2}{2} \cdot \delta m\right)^2}$$

Table 1: Velocities and Energies found for each of the soft guns.

Item	Velocity [m/s]	Δ Velocity [m/s]	Energy [J]	Δ Energy [J]
Spring Powered Hand Gun	63	1	0.51	0.02
Gas Powered Hand Gun	105	6	1.4	0.2
1st Rifle	190	10	8.9	0.9
2nd Rifle	323	5	25.8	0.8

The calculated values for velocities and energies of the soft guns and rifles are presented in Table 1. According to the values listed at the beginning, the energy of the spring powered hand gun is 0.49 J, which is within the margin of error of the measurements. The energy of the gas powered hand gun is given on it with the value of 1.6 J, which is also within the margin of error of the measurements.

2.2 Comments on the Choice of Equipment

Test runs for the experiment revealed that the container initially used had incorrect dimensions for the experiment. It was decided the containers to use were in the range of 50 to 70 liters, long and deep rather than wide, since in the case of the opposite, the pellet reached the wall or the bottom of the container too fast, not providing enough data points. After the test runs, I chose two containers to perform the experiment in (on Figure 1, the 50 liter container is filled with water on the table, and the 70 liter container is underneath the table). However, I shortly after have found out that the 50 liter container was too shallow and therefore I decided to only use the 70 liter container. I also found that in order for the pellet to be visible on the camera recording, the background of the container must be covered (in the cases where the pellets are white or silver). Initially, black plastic was used as a background, which was then changed to black paper taped to one side of the container.

Due to the fact that the Tracker program is used to process the data, I have added an object that determines the scaling of the video. I used a measuring stick, where 10 ± 0.1cm stripes were clearly marked (on Figure 1, the measuring is attached to the container filled with water).

In order to control the angle of the entry of the pellet, I used a stand for the soft guns (see Figure 1). The stand enabled changing the angles of entry and changing the height at which the soft guns were held. This resulted in the distance from the barrel to the point where the pellet hit the water changing slightly with every experiment. However, as mentioned before, the recording of the soft gun shooting a pellet through air shows that within approximately 1 meter, the speed of the pellet does not change in a significant way. Therefore it is fair to assume that the speed of the pellet immediately after it leaves the barrel is the same as immediately before the pellet hits the water.

Since the experiment is dealing with soft guns that shoot out pellets at high velocities, I have taken the following safety precautions: at all times while using the soft guns, I wore safety goggles and the soft guns were only shot pointing at the container with the liquid or the ground.

3 Results

3.1 Spring Powered Hand Gun

The first property I have decided to investigate closely is the entry angle of a pellet. I have first attempted to plot the raw data in one graph to see whether there were any trends that I could see right away (see Figures 5 and 6). However, I have found not only that there was too much scatter in the raw data for me to be able to see any trends, but also that the graphs looked nearly identical for all three different pellets. I then decided to fit a function through the data.

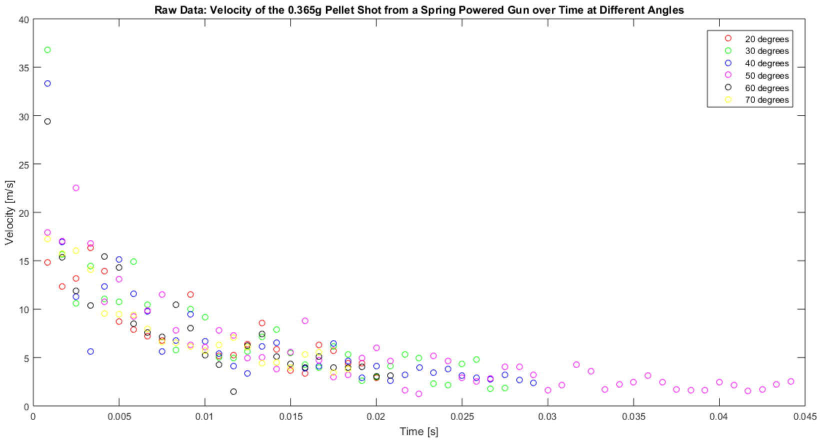


Figure 5: The graph representing raw data: the variation of velocity of the 0.365g green pellet shot out of the spring powered hand gun at various angles at water over time.

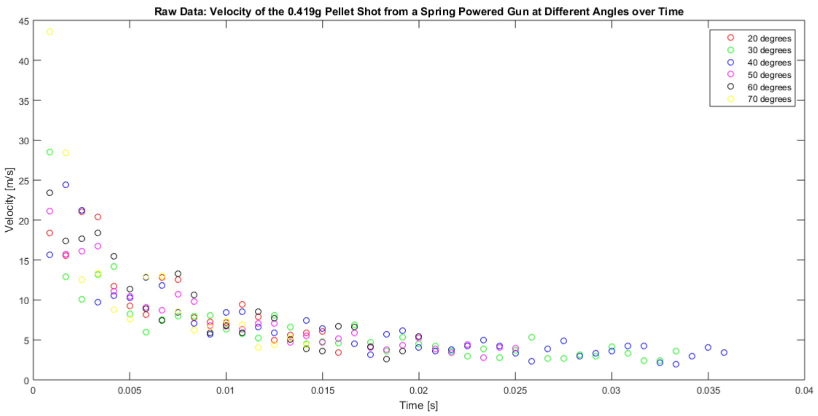
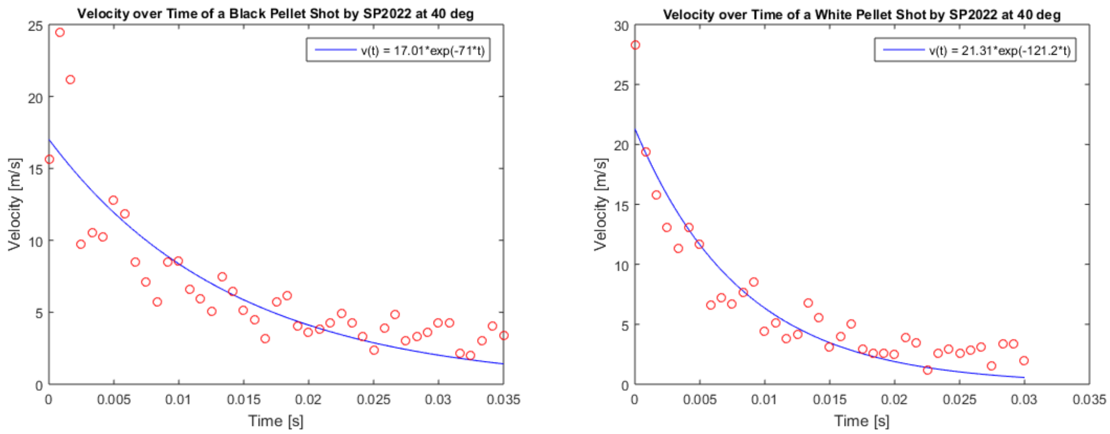


Figure 6: The graph representing raw data: the variation of velocity of the 0.419g black pellet shot out of the spring powered hand gun at various angles at water over time.

In order to compare the rates of the pellets' speed loss, I have decided to fit the data into an exponential function $v(t) = A_0 \cdot e^{B \cdot t}$, in which B is the time coefficient that gives me the information about how fast the speed is lost. The smaller the B, the faster the pellet lost its speed. The samples of the fits for black and white pellets are presented in Figures 7a and 7b respectively. I then wanted to investigate closer the variation of the time coefficient B over the mass of the pellets and the angles of entry. Therefore I have plotted the values of B over the angles of entry for the three pellets (Figure 8).

Fitting the data revealed a possible relation between the masses of the pellets and the loss of speed:

the 0.256g pellets have a smaller time coefficient consistently through angles 30 to 70, and the 0.419g pellets have the largest values for the time coefficient for angles 30, 40 and 60. The difference between B for the black and green pellets at 50 degree angle is fairly small, which might be a result of the fact that the mass of the black and green (0.365g) pellets have more similar values than the green and white or black and white pellets. The graph also shows that at angles 20 and 70, the patterns become more inconsistent. It is important to note that due to the limited volume of the container used, the amount of the data points varied with the angles, as at greater angles of entry the pellet hit the bottom of the container faster than at the smaller ones. As the variation of the B coefficient over the angles of entry of the pellet does not form into any clear pattern, it is most likely caused by the limited size and shape of the container.



(a) Black pellets (0.419g).

(b) White pellets (0.265g).

Figure 7: Exponential fits for black and white pellets shot out of the spring powered gun at water at 40 degrees entry angle.

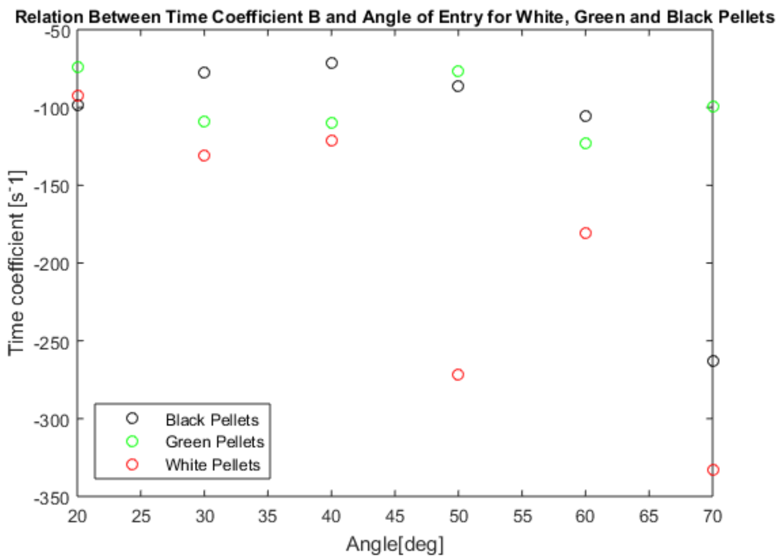


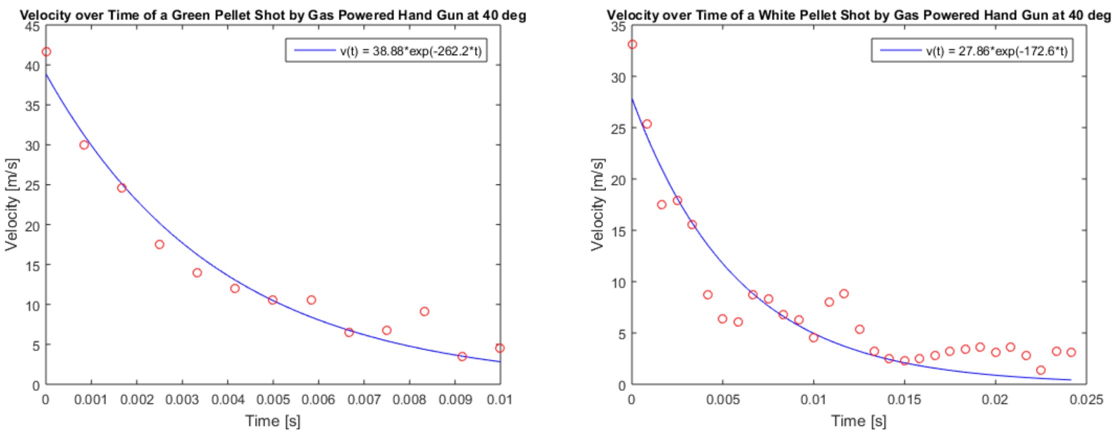
Figure 8: The variation of the time coefficient B over the angle of entry and the mass of a pellet.

I know that for high speed, the fluid resistance is $f = D \cdot v^2$ [3] for some constant D, where f is the drag force. Therefore the acceleration of the pellets should be proportional to $A \cdot v^2$ for some constant A that depends on the mass of the pellet. Due to that, I have decided to plot the acceleration of the pellets over their velocity and fit a function of the form $A \cdot v^2$ through the data, examples of which are presented in the Appendix C.

These fits were puzzling, as I expected the least massive, white pellets to decelerate the most at the highest velocities. It became apparent to me that the quality of the data might be worse than I had expected. I have decided to determine whether I could find any clearer trends with the data collected from the experiments with the gas powered hand gun. However, since I could not find any evidence for the angle of entry to have any influence on the rate of the speed loss of the pellets, I decided to only shoot the gas powered hand gun at the angles of entry of 40 and 50 degrees. These values were chosen due to the fact that the path of the pellet was not disturbed by the walls of the container the longest for these angles.

3.2 Gas Powered Hand Gun

Similarly to the spring powered hand gun, I have plotted the variation of the velocities of the pellets shot out of the gas powered gun and fitted a function $v(t) = A_0 \cdot e^{B \cdot t}$ through the data.



(a) Green pellets (0.365g). (b) White pellets (0.265g).

Figure 9: Exponential fits for green and white pellets shot out of the gas powered gun at water at 40 degrees entry angle.

Comparing these with Figures 7a and 7b, it becomes apparent that there are less data points collected for each shot when shooting with the gas powered gun. This is due to the fact that the speed of a pellet shot out of the gas powered gun is nearly three times as much energy as one shot out of the spring powered gun. Therefore, the pellets shot out of the gas powered gun reached the walls and the bottom of the container significantly faster than the ones shot out by the spring powered gun. The complete comparison of the time coefficient B values for the soft guns is presented in Table 2.

These results have again puzzled me, as the values for B for the gas powered gun seem to follow a different pattern than the ones for the spring powered one. The B values for the spring powered gun decrease as the mass of the pellet decreases, however in the case of the gas powered gun, the smallest B values are for the green, 0.365g pellets and the greatest are for the lightest, white pellets. I decided this must be a result of a large scatter of the data. I knew that some of the error resulted from the fact that the camera skips a frame every five frames. In Figure 2, it seems that the time it takes for the camera to skip one frame is double the time it takes for it to take one frame. Therefore it should be possible to correct the error by for each data set determining the first point in which the velocity is significantly greater than I expect it to be, then correcting that one data point by calculating the new velocity by:

$$v_x = \frac{x(i) - x(i-1)}{2 \cdot dt}$$

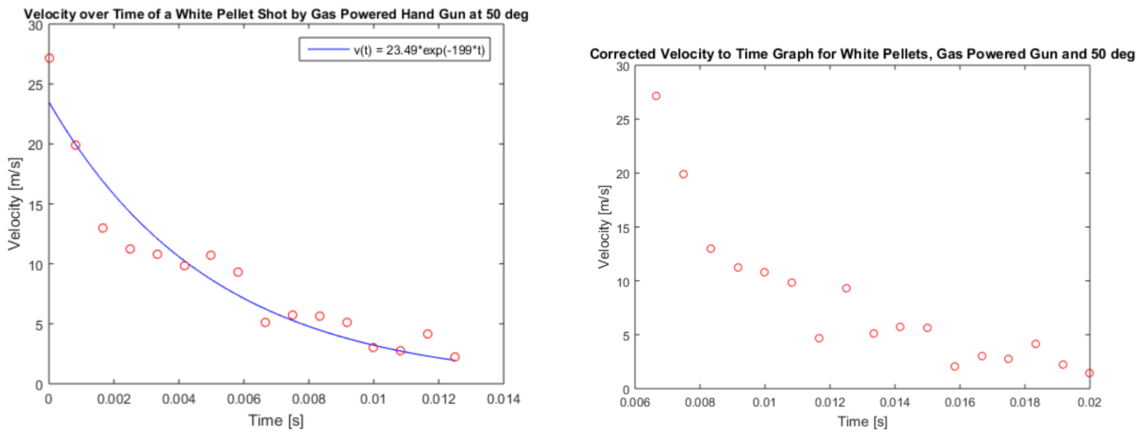
$$v_y = \frac{y(i) - y(i-1)}{2 \cdot dt}$$

$$v_{new} = \sqrt{v_x^2 + v_y^2}$$

where x, y describe the pellet's position, i is the number of the data point of which the velocity is larger than expected and $dt = \frac{1}{1200}s$. However, it turned out that finding the time of the skipped frame is more complex than I had thought. A sample of the corrected data plotted compared to the original data is presented in Figures 10b and 10a respectively.

Table 2: Time coefficient B values for the soft guns.

Spring Powered Hand Gun			Gas Powered Hand Gun		
Mass [m]	Angle [deg]	$B[s^{-1}]$	Mass [m]	Angle [deg]	$B[s^{-1}]$
0.256	40	-121.2	0.256	40	-172.6
0.365	40	-109.5	0.365	40	-262.2
0.419	40	-71.0	0.419	40	-214.9
0.256	50	-271.7	0.256	50	-199.0
0.365	50	-76.5	0.365	50	-341.9
0.419	50	-86.4	0.419	50	-257.5



(a) Original data.

(b) Corrected data.

Figure 10: The comparison of the plots of the corrected data to the original data for a 0.256g white pellet shot out of the gas powered gun at 50 degrees entry angle.

The first data point at which the velocity becomes significantly greater than the point before, is number 8. Hence, the seventh velocity point must have been corrected, and after that one, every fifth one until the end of the data. Hence I have corrected the 7th, 12th and 17th data points for the velocity. It can be easily seen from Figure 10b that the corrected data points do not seem to fit better within the data set, but become outliers. The conclusion to be drawn from this result is that finding the time it takes the camera to skip a frame is more complex than I expected.

As I was not able to correct for the error on the velocity measurements, I have decided to fit a function of the form $a(v) = A \cdot v^2$ through the data in order to be able to compare the data collected from shooting the two soft guns (showed on Figures 11 and 12).

3.3 Comparison of the Hand Guns

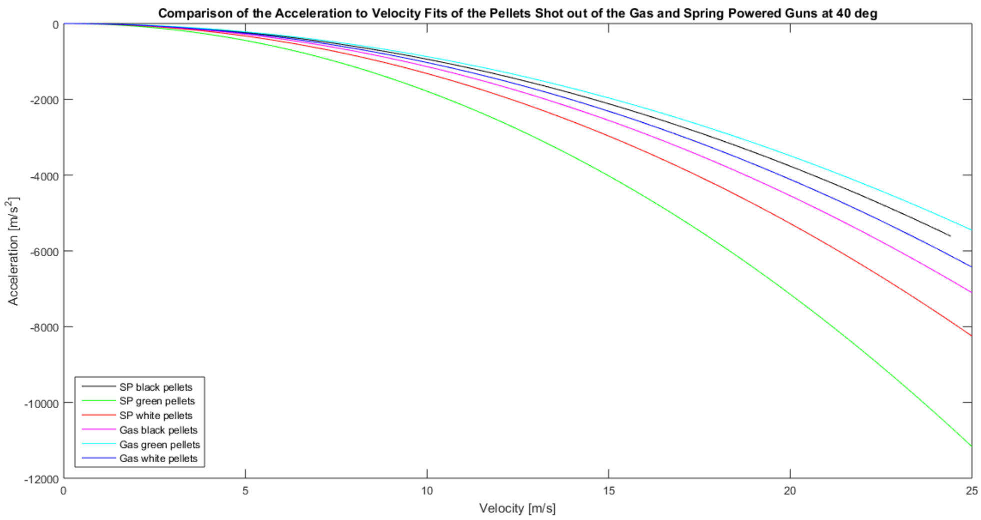


Figure 11: Comparison of the acceleration to velocity fits through the data collected by shooting all of the pellets from both of the hand guns at water at 40 degrees.

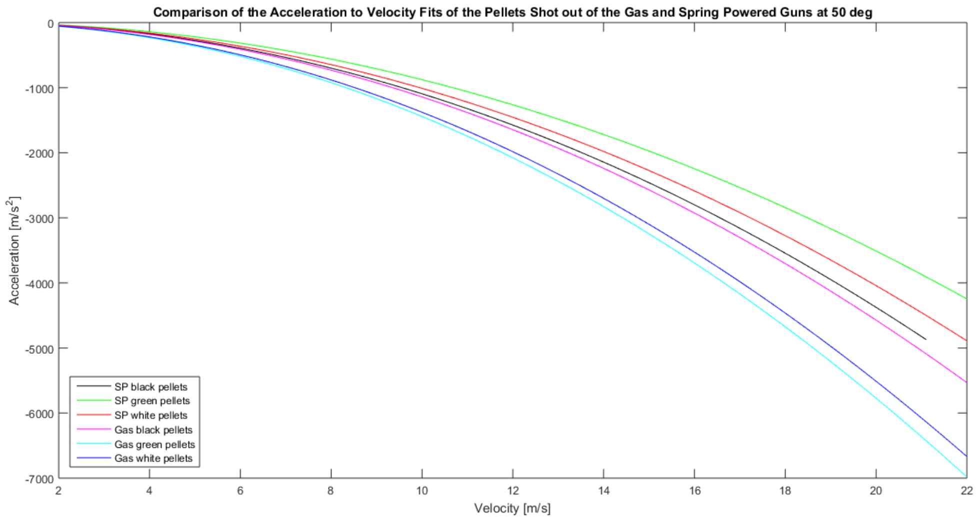


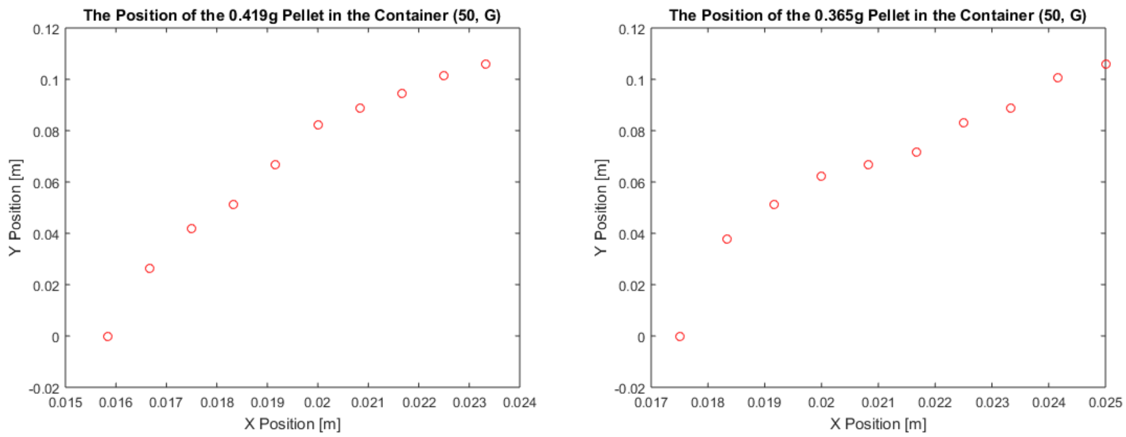
Figure 12: Comparison of the acceleration to velocity fits through the data collected by shooting all of the pellets from both of the hand guns at water at 50 degrees.

According to earlier mentioned drag equation, $f(v) = C \cdot v^2$ and $f(v) = a(v) \cdot m$ (Newton's Second Law), where m is the mass of the pellet. Then $a(v) = A \cdot v^2$ and A depends on the mass of the pellet. Therefore I have expected to see that for the different masses of the pellets, A would vary, but it would be very similar for a pellet of the same mass shot out of the two different hand guns. However, as the figures above show, this is not the case. Table 3 presents the values for the velocity coefficient A for all of the pellets shot out of each of the hand guns.

The results indicate that the data strays quite far from the accepted theory, therefore I have decided to try to figure out where the error of the data originates. I knew that the camera added extra dt every five frames, but I did not have a way to determine the value of it. I decided that looking at the raw data again might give me an idea of where the unusual behavior of the pellets might come from.

Table 3: Velocity coefficient A values for the soft guns.

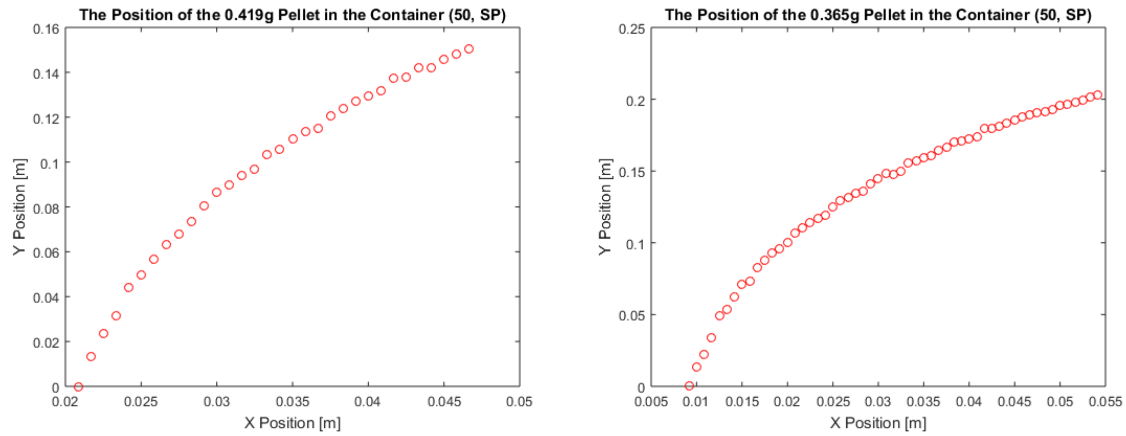
Spring Powered Hand Gun			Gas Powered Hand Gun		
Mass [m]	Angle [deg]	$A[m^{-1}]$	Mass [m]	Angle [deg]	$A[m^{-1}]$
0.256	40	-13.200	0.256	40	-10.280
0.365	40	-17.860	0.365	40	-8.726
0.419	40	-9.412	0.419	40	-11.360
0.256	50	-10.110	0.256	50	-13.780
0.365	50	-8.775	0.365	50	-14.430
0.419	50	-10.490	0.419	50	-11.430



(a) 50 degrees, black pellets, gas powered hand gun.

(b) 50 degrees, black pellets, gas powered hand gun.

Figure 13: Raw data: XY-position of the pellets.



(a) 50 degrees, black pellets, spring powered hand gun.

(b) 50 degrees, black pellets, spring powered hand gun.

Figure 14: Raw data: XY-position of the pellets.

It turns out that not only the amount of data collected from the motion of the pellets shot out by the gas powered gun is lesser than of the ones shot out by the spring powered one, but also the path of the pellet is more irregular. Some irregularities also appear on Figures 14a and 14b, however due to the fact that more data is collected in this case, it is easier to tell which of the data do not follow the overall trend of the path of the pellets. It is also important to note that for the clearness of the data, all of the starting points for the pellets have been set to $x, y, t = (0, 0, 0)$. However, in reality there was a slight difference between when the camera would catch the first glimpse of the pellet in the water and how far that was from the surface of the water in the container. Since the fits for $a(v) = A \cdot v^2$ depended so heavily on the first few data points where the velocities of the pellets were the greatest, that slight difference ended up playing a much bigger role than I have realized.

The pellets shot out by the rifles turned out to be too fast for the camera to collect more than three frames per each experiment and they barely lost any energy before they hit the bottom of the container. Therefore, due to the equipment limitations, I was not able to collect any data that could be conclusive. I ended up only being able to perform the experiments in water due to time constraints on the project.

4 Discussion

4.1 Theory

As mentioned in the Results section, when a body is experiencing the drag force while traveling through a fluid at high speeds, the magnitude of the force is [3]:

$$f = C \cdot v^2$$

where the constant C depends on the shape and size of the body and the density of the fluid, which is the source of idea to perform the experiment with different liquids. Therefore the deceleration of the pellet can be expressed as follows:

$$a = \frac{C}{m} \cdot v^2 = A \cdot v^2$$

Therefore, I know that the deceleration of the pellet in the water is dependent on its mass, but not its initial velocity. The smaller the value of the A constant (as A will be negative for a decelerating object), the greater the deceleration, and since $A = \frac{C}{m}$, then for smaller masses, the deceleration becomes greater.

Buoyancy is a concept describing the unbalanced upward force acting on the bottom of a body submerged in a liquid [4]. This is a result of the fact that fluid pressure increases with depth and the Pascal's principle, which states that "Pressure is transmitted undiminished in an enclosed static fluid" [5]. That unbalanced upward force is equal to the weight of the fluid displaced by the body, according to the Archimedes' principle [4].

Knowing this, I have decided to alter the angle of entry for the pellets. I thought that since the upward force acting on the pellets would change relative to the direction of the deceleration of the pellet, the angle of entry of the pellets would have an influence on the deceleration of the pellet.

4.2 Remarks on the Results

The results revealed that altering the angle of entry of the pellet did not have any significant influence on the deceleration of the pellet. As I have changed the angle of entry of the pellet, there was a variation in the time coefficient B and velocity coefficient A, however as the further examination of the data revealed, that was most likely a result of a large scatter of the data. Another indicator of this was the lack of a pattern that could be seen as the B and A coefficients varied with the angles (see Figure 8 and Table 3). It could be argued that the pattern could not be seen due to a large scatter of the collected data or a range of angles that was not wide enough. However, except for the buoyancy, there is no other theory that could back the possibility of the angles of entry to make a difference. Therefore it is fair to say that it is more likely that the entry angle does not influence the deceleration of the pellet.

The data describing the velocity of the pellets shot by a spring powered gun revealed a possible pattern related to the mass of the pellet: that is, the less massive the pellet, the quicker it loses speed (see Figure 8). This pattern did not continue when I found the values for the time coefficient for the pellets shot out of the gas powered gun (see Table 2). However, when I compared the raw data collected for the spring powered gun and the gas powered gun in Figures 13 and 14, it became apparent that the data collected by the spring powered gun had a greater chance to be more precise: not only was there more data points, but also the pellets shot out of the spring powered gun followed the expected path of their motion better than the pellets shot out of the gas powered guns. Moreover, as I know from the accepted theory, mass should influence the deceleration of the pellets and the smaller the mass of the pellets, the greater the deceleration.

According to the accepted theory, the deceleration of the pellets would not be influenced by their initial velocity, as it depends on the square of their instantaneous velocity. Therefore I have expected, when comparing the A coefficients of the fits through the data of the pellets shot out of the hand guns, that the velocity coefficients for the pellets of the same mass but shot out of different hand guns would be significantly closer in value than the ones of two different masses of the pellets. As can be seen in Table 3 and Figures 11 and 12, that was not the case. However, these values gave me a valuable insight into the error of the measurements and its source. As mentioned in the Method section, the fits $a(v) = A \cdot v^2$ have been heavily based on the first few data points for which the velocities have been the largest. However, since the distance from the surface of the water and the points at which the camera first shows the pellet varied slightly with each experiment, not all of the pellets were registered to have large velocities. For some pellets the first calculated velocity was between 40 and 45 m/s, and for some between 30 and 35 m/s (see Figures 9a and 9b). Therefore, the comparison of the two hand guns did not conclude anything about the influence of the initial velocity on the deceleration of the pellets, however it gave me an idea about the errors on the fits through the data.

4.3 The Limitations and the Improvement of the Experiment

One of the greatest limitations of this experiment was the equipment used. The high speed camera turned out to skip an unknown amount of time every five frames (see Figures 2 and 3). The camera's mechanism for taking the high speed video was made so it would divide the charge-coupled device into 5 separate pictures. However, when all of the five pictures were taken, the image had to be changed and I have no indication of how long that took. I tried making a script which would adjust the data, but I did it under the assumption that it took the exact same time to change the image as it took to capture one frame of the video. Moreover, the quality of the picture captured by the high speed camera was very low, which made it harder to track the pellets in the water. This limitation was made more significant by the fact that the Tracker program is precise to one pixel, which means that when the pellets were tracked, I had to aim precisely for the center of the pellet, which was often difficult to do due to the quality of the picture. Another limitation was caused by the amount of frames per second the camera registered: I could not use it for recording the motion of the pellets shot out of the rifles, as it could only register three or four different positions of the pellets in the water before they hit the bottom of the container. The amount of frames per second was also the reason for the error created by the previously mentioned variation in the position of the first data point registered relative to the surface of the water. Using a higher quality high speed camera would significantly improve the quality of the data collected in this experiment.

Another significant improvement for this experiment would be using a much larger, fully translucent container. The one that I have used was not fully translucent, which made it more difficult to track the motion of the pellets in the water. Moreover, the container used was only 27.0 ± 0.1 cm deep and 50.0 ± 0.1 cm wide. This resulted in the pellets hitting the bottom and the walls of the container before they have lost a significant amount of their speed, which resulted in incomplete data. A larger container would also likely make it possible to perform the experiment with the rifles, as the pellets shot out of them could decelerate before hitting the bottom.

Time management and time constraint were also significantly limiting for this project. I have planned to alter the angle of entry, the mass and the initial velocity of the pellet as well as the liquid used and its temperature. I have quickly realized that I could not perform all of these experiments within the time given for the project, hence I decided not to alter the temperature of the liquid. However, it turned out I ended up not having enough time to alter the type of the liquid either. A way to improve this experiment would be to include the variation of the type of the fluid used, as I know from the accepted theory that the velocity coefficient A depends on the density of the fluid. Altering the temperature of the fluid used could also give interesting results, as the density and the viscosity of the fluid are related to each other and the viscosity of the fluid is altered with its temperature.

In order to improve the quality of the $a(v) = A \cdot v^2$ fits, I could have taken more recordings for each of the experiments. This would increase the amount of the data corresponding to the greater velocities and make the fit more accurate. Moreover, the error caused by the variation in the distance from the first data point to the surface of the water would significantly decrease.

5 Conclusion

For a body moving through water at a high speed, it is only its mass and shape that will influence its deceleration. The rate of its speed loss will be greater the smaller its mass. The entry angle of the body will not have an influence on the body's deceleration, and neither will its initial speed. Only two of those trends have been possible to determine through this experiment: the deceleration's dependence on the mass and its independence on the entry angle of the body. Moreover, it has been determined that the of speed of the body over time can be described by a function of the form $v(t) = A \cdot e^{B \cdot t}$ and the deceleration of the body by $a(v) = A \cdot v^2$. The deceleration of the body is caused by the drag forces acting on it which is given by the drag equation $f(v) = C \cdot v^2$, where C depends on the density of the fluid that the body is moving through. Therefore, for a body moving through a fluid at a high speed, the greatest deceleration will be given in a situation where the body has a small mass and is traveling through a very dense fluid.

6 Acknowledgments

First and foremost, I would like to thank my supervisor Ian Bearden for the support and guidance that he has provided throughout this project. Moreover, a great deal of the knowledge and skill required for this project was available to me due to the great efforts of the Copenhagen University's Physics Lab Team lead by Ian Bearden and Børge Svane Nielsen, and so I would like to thank them for guiding me during the Lab Classes in Blocks 1 and 2. This investigation could not have happened without the equipment that was provided for me by the Niels Bohr Institute, of which I am also very grateful. Lastly, I would like to thank Jonas Martin Søndermølle, who took his time to proof-read this report in the context of the coherency and clearness of the language used.

A Appendix A
The Setup for Determining Rifles' Velocities

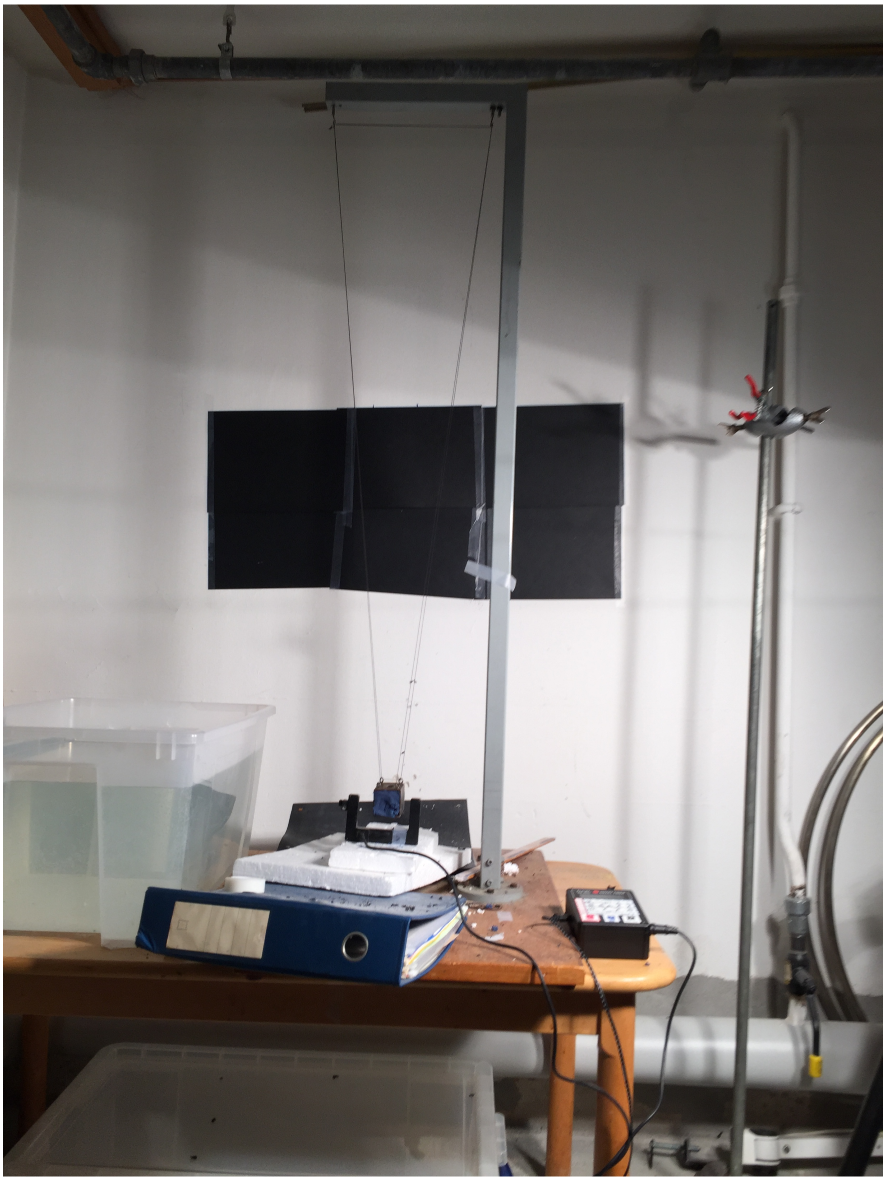


Figure 15: The pendulum that was shot at by the rifles and the motion of was recorded and analyzed in order to find the energies of the rifles.

Figure 16: Two frames layered on top of each other, one showing the pendulum in the equilibrium position and the other showing it at maximum y-displacement.

B Appendix B
The Derivation of the Velocity Formula for the Rifles

Once a pellet is shot out of one of the rifles, it starts moving with a certain initial velocity, until it hits the pendulum and undergoes complete elastic collision, causing the pendulum to swing. Because of the conservation of momentum, $\Delta p = 0$, and so

$$m \cdot v_1 = (m + M) \cdot v_2$$

where m is the mass of the pellet, M is the mass of the pendulum, p is momentum, v_1 is the initial speed of the pellet and v_2 is the speed of the pendulum and the pellet right after the collision. As the pendulum starts to swing, the conservation of mechanical energy is assumed, hence $\Delta K + \Delta U = 0$, where K is the kinetic energy and U is the potential energy of the pendulum and the pellet. Since Δh, the difference in the height of the pendulum's swing, is 0 initially, and at the highest point of the swing pendulum's velocity is 0, I have

$$\frac{1}{2} \cdot m \cdot v_2^2 = m \cdot g \cdot \Delta h$$

$$v_2 = \sqrt{2 \cdot g \cdot \Delta h}$$

From the first equation:

$$v_2 = \frac{m \cdot v_1}{m + M}$$

$$\frac{m \cdot v_1}{m + M} = \sqrt{2 \cdot g \cdot \Delta h}$$

$$v_1 = \sqrt{2 \cdot g \cdot \Delta h} \cdot \frac{m + M}{m}$$

C Appendix C

Examples of $A \cdot v^2$ Fits for the Acceleration to Velocity Graphs

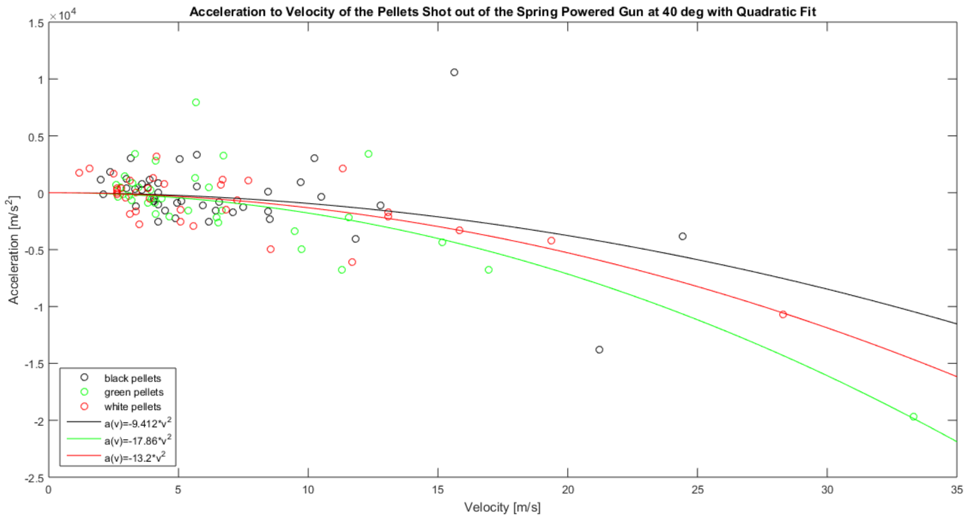


Figure 17: Acceleration to velocity of the pellets shot out of the spring powered hand gun at 40 degrees entry angle with a fit of a curve of the form $A \cdot v^2$ for 0.256g white, 0.365g green and 0.419g black pellets.

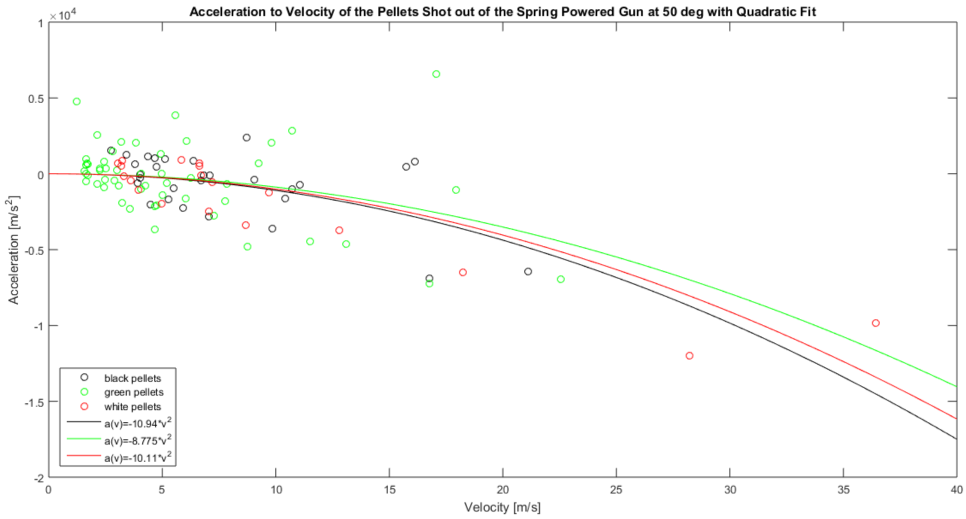


Figure 18: Acceleration to velocity of the pellets shot out of the spring powered hand gun at 50 degrees entry angle with a fit of a curve of the form $A \cdot v^2$ for 0.256g white, 0.365g green and 0.419g black pellets.

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