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Latex Assignment #1

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Consider the matrix given below.

$$A = \begin{bmatrix} 1 & 2 & 4 & 0 \\ -3 & 1 & 5 & 2 \\ -2 & 3 & 9 & 2 \end{bmatrix}.$$

1. The transformation associated with A maps $\mathbb{R}^4 \to \mathbb{R}^3$.

2. Describe the row space of A.

   Solution: The row space of A is the subspace of $\mathbb{R}^n$ spanned by its rows, or the collection of all the linear combinations of the rows of A. When we reduce A, we find that

   $$R = \begin{bmatrix} 1 & 0 & \frac{-6}{7} & \frac{-4}{7} \\ 0 & 1 & \frac{17}{7} & \frac{2}{7} \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

   Here, we can see that the last row of R is all zeros, meaning the last row of A is a linear combination of the first two rows of A. The linear combination of the first two rows of A is therefore our row space, and can be expressed as

   $$\text{rowspace(A)} = \left\{ a \begin{bmatrix} 1 & 2 & 4 & 0 \end{bmatrix} + b \begin{bmatrix} -3 & 1 & 5 & 2 \end{bmatrix} \;\middle|\; a, b, c, d \in \mathbb{R} \right\}$$

3. Describe the column space of A.

   Solution: The column space of A is the subspace of the columns of A, or the collection of all linear combinations of the columns of A. We can see from R that the first and second comlumns are linearly independent, therefore the first two columns of A are independent and constitute the column space of A, or

   $$\text{col(A)} = \left\{ a \begin{bmatrix} 1 \\ -3 \\ -2 \end{bmatrix} + b \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix} \;\middle|\; a, b \in \mathbb{R} \right\}$$

4. What is the Rank of A?

   Solution: The rank of A is the dimension of the row space and column space; that is, the maximum number of independent rows or columns. As we can see from both the row and column spaces, that number is 2. Therefore, Rank(A) = 2

5. What is the Nullity of A?

   Solution:The nullity of A is the dimension of the null space of A. The number of columns n equals the rank r plus the nullity. Thus nullity = n - r = 4 - 2 = 2

6. Describe the null space of A

   Solution: The null space space of A is the collection of vectors x for which $Ax = 0$. We can use the reduced row echelon form R of the matrix A to find the basis vectors for the nullspace of A. Call these basis vectors n_1 and n_2 and let the matrix N have n_1 and n_2 as columns.The identity matrix fills in the remaining rows associated with the pivot variables.

$$R = \begin{bmatrix} 1 & 0 & \frac{-6}{7} & \frac{-4}{7} \\ 0 & 1 & \frac{17}{7} & \frac{2}{7} \\ 0 & 0 & 0 & 0 \end{bmatrix} \Longrightarrow N = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ \frac{6}{7} & \frac{4}{7} \\ \frac{-17}{7} & \frac{-2}{7} \end{bmatrix}$$