Theory of Computation - University of Texas at El Paso

On the Complexity of Dealing with Inconsistency in Description Logic Ontologies

Riccardo Rosati Universitá di Roma, Italy

Ismael Villanueva-Miranda | Final Project
ivillanueva5@miners.utep.edu

May 4, 2017

Outline

An ontology defines a common vocabulary for researchers who need to share information in a domain.

EXAMPLE: PIZZA ONTOLOGY

CONCEPTS
Pizza, Pizza Base, Pizza Toppings

PROPERTIES
hasBase, hasCheeseTopping, hasCheeseTopping, HasPeperonniTopping

Pizza hasBase DeepPanBase and hasCheeseTopping Mozzarella

What are Description Logics (DL)?

A Description Logics (DL) models concepts, roles and individuals, and their relationships.

- DL are a family of formal knowledge representation languages.
- DL are decidable fragments of First Order Logic

$$
\begin{aligned}
\mathsf{VegetarianPizza} &\sqsubseteq \mathsf{Pizza} \\
\mathsf{MagheritaPizza} &\sqsubseteq \mathsf{Pizza} \\
\mathsf{TomatoTopping} &\sqsubseteq \mathsf{VegetableTopping} \\
\mathsf{MozzarellaTopping} &\sqsubseteq \mathsf{CheeseTopping} \\
\mathsf{VegetarianPizza} &\equiv \forall\, \mathsf{hasTopping}\ (\mathsf{VegetableTopping} \sqcup \mathsf{CheeseTopping}) \\
\mathsf{MagheritaPizza} &\sqsubseteq \exists\, \mathsf{hasTopping}\ \mathsf{MozarellaTopping} \sqcap \\
&\quad \exists\, \mathsf{hasTopping}\ \mathsf{TomatoTopping} \sqcap \\
&\quad \forall\, \mathsf{hasTopping} \\
&\qquad (\mathsf{MozzarellaTopping} \sqcup \mathsf{TomatoTopping})
\end{aligned}
$$

Logical consequence: $\mathsf{MagheritaPizza} \sqsubseteq \mathsf{VegetarianPizza}$

An inconsistency is identified with the existence of unsatisfiable axioms[1].

$$\textbf{VegetarianPizza} \sqsubset \forall \text{ hasTopping Vegetable}$$
$$\textbf{VegetarianPizza} \sqsubset \exists \text{ hasTopping Meat}$$

[1] Axioms: assertions in a logical form that together comprise the overall theory that the ontology describes in its domain of application.

The problem: Dealing with inconsistency

- The size of ontologies used by real applications is scaling up
- Ontologies are increasingly merged and integrated into larger ontologies
- The probability of introducing inconsistency is consequently getting higher
- Dealing with inconsistency is becoming a practical issue in ontology-based systems

The Solution: Alternative Approach

An alternative approach is to define inconsistency-tolerant semantics:

- are able to derive meaningful conclusions from inconsistent ontologies
- can be the formal basis for an automated treatment of inconsistency
- are based on repairing (i.e., modifying) the extensional knowledge (ABox)
- a repair is a maximal subset of the ABox that is consistent with the TBox

The DLs mainly considered in this paper are the following:

- EL: A prominent tractable DL
- ALC: A very well-known DL which correspond to multimodal logic K_n
- SHIQ: A very expensive DL which constitutes the basis of the OWL family

DL	concept and role expressions	TBox axioms
$\mathcal{EL}_\bot$	$C ::= A \mid \bot \mid C_1 \sqcap C_2 \mid \exists P.C$ $R ::= P$	$C_1 \sqsubseteq C_2$
$\mathcal{ALC}$	$C ::= A \mid C_1 \sqcap C_2 \mid \neg C \mid \exists P.C$ $R ::= P$	$C_1 \sqsubseteq C_2$
$\mathcal{SHIQ}$	$C ::= A \mid \neg C \mid C_1 \sqcap C_2 \mid (\geq n\, R\, C)$ $R ::= P \mid P^-$	$C_1 \sqsubseteq C_2$ $R_1 \sqsubseteq R_2$ $\mathrm{Trans}(R)$

The Solution: Data Complexity

In this paper, they consider data complexity (i.e., the complexity with respect to the size of the ABox) and combined complexity (i.e., the complexity with respect to the size of the whole input) of UCQ[2] entailment and instance checking.

DL (problem)	data complexity	combined complexity
$\mathcal{EL}, \mathcal{EL}_\bot$ (IC)	PTIME	PTIME
$\mathcal{EL}, \mathcal{EL}_\bot$ (UCQ)	PTIME	NP
$\mathcal{ALC}$ (IC)	coNP	EXPTIME
$\mathcal{ALC}$ (UCQ)	coNP	EXPTIME
$\mathcal{SHIQ}$ (IC)	coNP	EXPTIME
$\mathcal{SHIQ}$ (UCQ)	coNP	2-EXPTIME

Table: results on the complexity of instance checking and UCQ entailment under standard semantics in the DLs

[2] Union of Conjunctive Queries

Complexity of UCQ entailment over DL KBs under inconsistency-tolerant semantics.

	data complexity				combined complexity			
semantics	AR	CAR	IAR	$ICAR$	AR	CAR	IAR	$ICAR$
$\mathcal{EL}_\bot$ (IC)	coNP	DP	coNP	DP	coNP	DP	coNP	DP
$\mathcal{EL}_\bot$ (UCQ)	coNP	$\Delta_2^p[\mathcal{O}(\log n)]$	coNP	$\Delta_2^p[\mathcal{O}(\log n)]$	Π_2^p	Π_2^p	$\Delta_2^p[\mathcal{O}(\log n)]$	$\Delta_2^p[\mathcal{O}(\log n)]$
$\mathcal{ALC}$ (IC)	Π_2^p	$\mathrm{BH}_2(\Sigma_2^p)$	Π_2^p	$\mathrm{BH}_2(\Sigma_2^p)$	ExpTime	ExpTime	ExpTime	ExpTime
$\mathcal{ALC}$ (UCQ)	Π_2^p	$\Delta_3^p[\mathcal{O}(\log n)]$	Π_2^p	$\Delta_3^p[\mathcal{O}(\log n)]$	ExpTime	ExpTime	ExpTime	ExpTime
$\mathcal{SHIQ}$ (IC)	Π_2^p	$\mathrm{BH}_2(\Sigma_2^p)$	Π_2^p	$\mathrm{BH}_2(\Sigma_2^p)$	ExpTime	ExpTime	ExpTime	ExpTime
$\mathcal{SHIQ}$ (UCQ)	Π_2^p	$\Delta_3^p[\mathcal{O}(\log n)]$	Π_2^p	$\Delta_3^p[\mathcal{O}(\log n)]$	2-ExpTime	2-ExpTime	2-ExpTime	2-ExpTime

The authors conclude the following:

- reasoning under the approximated semantics is in general intractable even for tractable DLs.
- reasoning under the inconsistency-tolerant semantics is inherently intractable, even for very simple DLs.