

# Resolução 3.9

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Guilherme Baumgratz Figueiroa

Universidade Federal de Ouro Preto

1. Prove os seguintes sequentes usando dedução natural:

b)  $\{ \forall x. P(x) \rightarrow \neg Q(x) \} \vdash \neg \exists x. P(x) \wedge Q(x)$

$$\overline{\neg \exists x. P(x) \wedge Q(x)}$$

b)  $\{ \forall x. P(x) \rightarrow \neg Q(x) \} \vdash \neg \exists x. P(x) \wedge Q(x)$

$$\frac{\perp}{\neg \exists x. P(x) \wedge Q(x)} \text{ (Contr)}$$

b)  $\{ \forall x. P(x) \rightarrow \neg Q(x) \} \vdash \neg \exists x. P(x) \wedge Q(x)$

$$\frac{\frac{Q(b) \quad \neg Q(b)}{\perp} \text{ (And-I)}}{\neg \exists x. P(x) \wedge Q(x)} \text{ (Contr)}$$

b)  $\{ \forall x. P(x) \rightarrow \neg Q(x) \} \vdash \neg \exists x. P(x) \wedge Q(x)$

$$\frac{\frac{I}{Q(b)} () \quad \frac{II}{\neg Q(b)} ()}{\perp} (And-I)$$

$$\frac{\perp}{\neg \exists x. P(x) \wedge Q(x)} (Contr)$$

b)  $\{ \forall x (P(x) \rightarrow \neg Q(x)) \} \vdash \neg \exists x. P(x) \wedge Q(x)$

$\overline{Q(b)}$  (I)

b)  $\{ \forall x. P(x) \rightarrow \neg Q(x) \} \vdash \neg \exists x. P(x) \wedge Q(x)$

$$\frac{P(b) \wedge Q(b)}{Q(b)} \text{ (And-E)}$$

b)  $\{ \forall x. P(x) \rightarrow \neg Q(x) \} \vdash \neg \exists x. P(x) \wedge Q(x)$

$$\frac{\frac{\frac{\exists x. P(x) \wedge Q(x)}{P(b) \wedge Q(b)} \text{ (Exist-E)}}{Q(b)} \text{ (And-E)}}{Q(b)}$$

b)  $\{ \forall x (P(x) \rightarrow \neg Q(x)) \} \vdash \neg \exists x. P(x) \wedge Q(x)$

$$\overline{\neg Q(b)} \quad ()$$

b)  $\{ \forall x (P(x) \rightarrow \neg Q(x)) \} \vdash \neg \exists x. P(x) \wedge Q(x)$

$$\frac{P(b) \rightarrow \neg Q(b) \quad P(b)}{\neg Q(b)} \text{ (Imp-E)}$$

b)  $\{ \forall x.P(x) \rightarrow \neg Q(x) \} \vdash \neg \exists x. P(x) \wedge Q(x)$

$$\frac{\frac{\forall x.P(x) \rightarrow \neg Q(x)}{P(b) \rightarrow \neg Q(b)} \text{ (All-E)} \quad P(b)}{\neg Q(b)} \text{ (Imp-E)}$$

b)  $\{ \forall x.P(x) \rightarrow \neg Q(x) \} \vdash \neg \exists x. P(x) \wedge Q(x)$

$$\frac{\frac{\forall x.P(x) \rightarrow \neg Q(x)}{P(b) \rightarrow \neg Q(b)} \text{ (All-E)} \quad \frac{P(b) \wedge Q(b)}{P(b)} \text{ (And-E)}}{\neg Q(b)} \text{ (Imp-E)}$$

b)  $\{ \forall x.P(x) \rightarrow \neg Q(x) \} \vdash \neg \exists x. P(x) \wedge Q(x)$

$$\frac{\frac{\forall x.P(x) \rightarrow \neg Q(x)}{P(b) \rightarrow \neg Q(b)} \text{ (All-E)} \quad \frac{\frac{\exists x.P(x) \wedge Q(x)}{P(b) \wedge Q(b)} \text{ (Exist-E)} \quad \frac{P(b) \wedge Q(b)}{P(b)} \text{ (And-E)}}{P(b) \rightarrow \neg Q(b) \quad \frac{P(b) \wedge Q(b)}{P(b)} \text{ (And-E)}}{\neg Q(b)} \text{ (Imp-E)}$$

b)  $\{ \forall x.P(x) \rightarrow \neg Q(x) \} \vdash \neg \exists x. P(x) \wedge Q(x)$

$$\begin{array}{c}
 \frac{\frac{\frac{\exists x.P(x) \wedge Q(x)}{P(b) \wedge Q(b)} \text{ (And-E)}}{Q(b)} \text{ (And-E)}}{\frac{\frac{\frac{\exists x.P(x) \wedge Q(x)}{P(b) \wedge Q(b)} \text{ (And-E)}}{Q(b)} \text{ (And-E)}}{\perp} \text{ (Contr)}}{\neg \exists x.P(x) \wedge Q(x)} \text{ (Contr)} \\
 \frac{\frac{\frac{\exists x.P(x) \wedge Q(x)}{P(b) \wedge Q(b)} \text{ (And-E)}}{Q(b)} \text{ (And-E)} \quad \frac{\frac{\forall x.P(x) \rightarrow \neg Q(x)}{P(b) \rightarrow \neg Q(b)} \text{ (All-E)} \quad \frac{\frac{\frac{\exists x.P(x) \wedge Q(x)}{P(b) \wedge Q(b)} \text{ (And-E)}}{P(b)} \text{ (And-E)}}{\neg Q(b)} \text{ (Imp-E)}}{\neg Q(b)} \text{ (And-I)}
 \end{array}$$