What is the maximum altitude reached by a Superpressure balloon? Can we control the balloons altitude with an air pump?

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Abstract

A detailed report of findings on the altitudes which can be reached by super pressure balloons and how various factors and considerations affect this. Superpressure balloons are deployed and researched by various organisations including NASA, to solve technical limitations such as cell tower coverage as well as advancing fields of research. Balloons are used in planetary exploration, and weather prediction to teaching primary school physics. The versatile yet simple aerostat has been a valuable tool in many areas of engineering and their altitude ceiling is of great scientific interest. To solve the problem without the ability to physically reproduce the scenario, required mathematical models to be created as a means of simulating the effects of real world physics. A degree great enough to output an accurate and hence useful result without becoming too complex to be computable is the fine balance attempted to be created by this paper.

Introduction

A super pressure balloon is a sealed flexible bag filled with a light gas allowing it to take off and float within the atmosphere. Made of a canvas capable of withstanding high tensile loads whilst remaining inextensible and flexible, they will not stretch, due to internal pressures. So the internal volume will not increase past its maximum once fully inflated. NASA frequently uses them for extended duration unmanned flights, with the objective of high altitude scientific data gathering. This is due to their ability to maintain a constant altitude regardless of changes in external factors.

Parameters

A mathematical model needs to outline the variables which have an effect on the balloon and whether they need to be taken into consideration. It is given the balloon has a diameter of 15 metres and is filled with pure helium gas.

The Earth's atmosphere and the helium within the balloon behave as ideal gases. This allows Boyles, Charles and Gay-lussacs laws to be applied. Given the gases will not be placed under any extreme circumstances such as ultra-high temperatures or pressures, this is a reasonable assumption to make giving an accurate description of how the gas molecules will behave.
The balloon operates in optimum conditions with no atmospheric turbulence or other adverse weather effects.
The air has constant humidity throughout the atmosphere from sea level to the thermosphere due to the effect changing water vapour content has on air density, which will be explained in more detail later on.
The suns light rays incident with the balloon causes no transfer of energy to the canvas or internal gas. This is because the heating effect would cause changes to the Helium's internal energy; this is difficult to accurately model, requiring more advanced thermodynamics and physics. The effect would be nominal at best and so incorporating it into the model would make very minimal improvement in the accuracy of the solution.
Initially assuming the balloon to be perfectly spherical allows geometric equations to be applied to calculate internal volume and surface area. Whilst being a reasonable assumption most Super Pressure Balloons adopt a pumpkin shape for structural stability. Despite a lack of data on the exact shape of balloons in flight; a more accurate representation of its shape is found using an oblate spheroid. This allows for an improvement in accuracy for a small increase in complexity.

Structure of Balloon

NASA create Super pressure balloons out of Co-Extruded Linear Low Density Poly-Ethylene Film (LLDPE). Co-Extrusion is a method of layering multiple molten polymers to produce a product with desired characteristics, which a single polymer cannot achieve. The linearity of the polymer gives it improved rheological qualities over standard Polyethylene. This is due to the short chained molecular branches formed using a specialist process. It can withstand higher shear and stress forces whilst remaining incredibly light. For example, LLDPE with a density of $915kgm^{-3}$ can withstand 30MPa of tensile stress. Assume the balloon to be a perfect sphere when fully inflated, giving it an internal volume and surface area of:

$$V = \frac{4}{3}\pi r^3 = 1767m^3$$

$$S_A = 4\pi r^2 = 707m^2$$

However, NASA state that the balloons are not spherical when inflated. With a polar radius equal to 60 percent of their equatorial radius, their shape more closely resembles that of an oblate spheroid. Whilst this is an improvement to the model, it is not perfect. Given that the exact shape of the balloon is more pumpkin like, to model the divots much more detailed measurements of the canvas are needed and use calculus to further approximate the volume and surface area. Doing so would yield very little increase in model accuracy as the uncertainty is already very low. Taking the 15 metre diameter reading to be measured from the balloons furthest points, gives the balloon a height of 9 metres. Using these measurements the volume and surface area can be found:

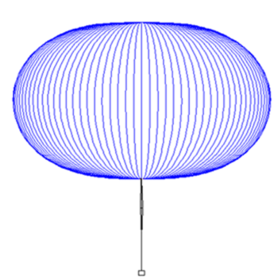

Figure 1: NASA's Depiction Of the balloon's inflated shape.

$a = equatorial radius, r.$ $b = polar radius,$ $\frac{3}{5}r,$ $r = 7.5m.$

$$V = \frac{4}{3}\pi a^2 b$$

$$V = \frac{4}{3}\pi r^2 \frac{3}{5} r = 1060m^3$$

$$S_{Oblate} = 2\pi a^2(1 + \frac{1 - e^2}{e}\tanh^{-1}(e)), e^2 = 1 - \frac{b^2}{a^2}$$

$$e^2 = 1 - \frac{\frac{3}{5}r^2}{r^2} = \frac{2}{5}$$

$$S_{Oblate} = 2\pi r^2(1 + \frac{1 - e^2}{e}\tanh^{-1}(e))$$

$$S_{Oblate} \approx 528m^2$$

Dry Mass M_o

The density of LLDPE is $915kgm^{-3}$. NASA states their balloons' canvas is 1.5mils thick.

Sub note: 1 mil is 1 thousandth of an inch and not a millimetre, thanks to Dr Michael McCann for clarifying NASAs strange American measuring system.

1.5mil= 38.1 micrometres.

To cover balloon's canvas area of $528m^2$; a volume of $0.02m^3$ is required.

$528m^2 \times 38.1 \times 10^{-3} = 0.02m^{-3}$

Which will have a mass of:

$V \times \rho = M$

$0.02 \times 915 = 18.3kg$

The balloon will also carry a payload in order for it to have any sort of scientific value. This could be a transmitter, data logger or camera; potentially to capture the Earths curvature (or anything that the balloons operators would want to research). I have decided to give the balloon a payload of 11.7kg, consisting of a launching mechanism and a small air pump in order to address the secondary question which is; can we control the balloons altitude with an air pump. Finding reliable data on the mass of such a device is difficult but the value given is within an appropriate range.

The dry mass of the balloon [fully deflated] = Mo = 30kg.

Lifting Gas Considerations

Initially assumptions were that the balloon would be fully inflated to the outside pressure at take-off. Being at sea level this would be standard pressure and temperature, defined as 273.15 Kelvin and 100,000Pascals. Given these factors, the amount of helium within the balloon would be:

$$\frac{PV}{RT} = n$$

$$\frac{10^5 \times 1060}{8.31 \times 273} = 46724$$

46724 moles of the monoatomic gas would be required.

Helium has an atomic mass of 4, so this amount of helium would have a total mass of:

$n \times A_r = M_h$

$M_h = 46724 \times 4 = 187kg$

Thus giving a take off mass of:

$M_o + M_h = 187 + 30$

$= 217kg$

The volume of dry air displaced by this balloon is equivalent to that of the balloon; given air has a molecular mass of 29 at 0 percent humidity:

46724 moles would have a mass of

$n \times M_r = M_a$

$46724 \times 29 = 1355kg$

Sub note: Increasing the airs humidity will make the air less dense and so the air displaced would have a lower mass. However, the variation in mass is minute due to the small difference in molecular mass, the small relative volume of the balloon and the small change in water vapour content of dry to high humidity air (typically 5 percent). Air is considered dry in the International Atmospheric model so this is a fair assumption to make.

Due to the large difference in mass of the inflated balloon and displaced air the balloon has positive buoyancy and will lift off when released.

$$(M_a - (M_o + M_h)) \times g = 11.152 kN (of positive buoyancy.)$$

$g = gravitational field strength = 9.81 N kg^{-1}$

However, an improved version of the balloon would take advantage of the fact that the balloon is excessively buoyant at sea level. Reducing the initial volume of gas put in the balloon can reap significant weight reductions and also greatly reduce the internal pressure of the balloon. This is because filling the in-expandable balloon to its maximum volume will cause the helium atoms will have to disperse much more. The result is: much higher altitudes can be reached before equilibria is found between the external air density and the balloon systems mean density and as such, has become neutrally buoyant and reached its maximum altitude. The optimum solution to this can be shown as a linear inequality.

To achieve positive buoyancy:

$$M_h + M_o < M_a$$

Under constant temperature and pressure, a mole of gas will displace the same mass, so it can be said that:

$$4n + 30,000 = 29n$$

$$30,000 = 25n$$

$$n = 1200$$

This means that at sea level the balloon filled with 1200 moles of helium is neutrally buoyant. Being raised a small degree will generate enough buoyancy for take-off to begin. This much helium has a mass of 4.8kg and so the total balloon systems mass at takeoff is 34.8kg.

Aproximating Atmospheric Pressure

The atmospheric pressure is the easiest variable to model. Tables of data show Appendix 1) atmospheric pressure, beginning at 10^5 Pascals (1 Bar) at sea level, dropping exponentially to almost 0 Pascals at 40km above sea level. The data very closely follows the function:

$$P(h) = P_o e^{-(\frac{h}{h_o})}$$

Where $P_o = 100,000Pa$ and $h_o = 7000m$

Graphical representation of this relationship:

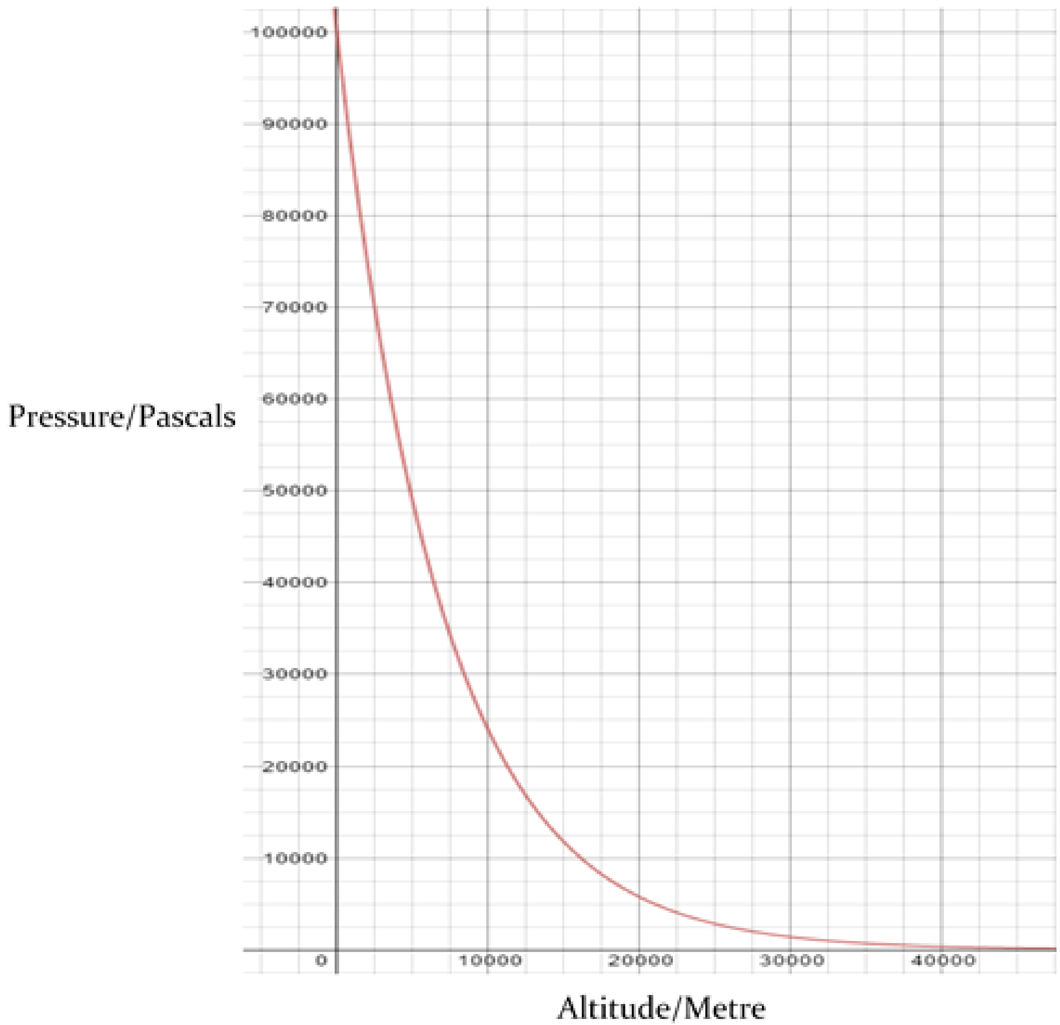

Figure 2: Created with Desmos graphical calculator

Atmospheric Temperature Lapse Rate

The relationship quantifying how the rate of change of air temperature varied with respect to altitude is much more difficult to find. Most sources agree the rate is linear, following the differential form:

$$-\gamma = \frac{\delta T}{\delta h}$$

However, it has been stated this constant, gamma, held true throughout the atmosphere from sea level to the thermosphere, values being given of around $6.5Kkm^{-1}$. or that different lapse rates occur at separate layers of the altitudes. The most accurate descriptions showed the rate fluctuated from positive to negative between atmospheric layers. These graphs gave the most resolute description of the relationship and seem to be congruent with other reliable sources.

Using geometry a lapse rate for the stratosphere can be generated. The stratospheric lapse rate comes in the form:

$$T(h) = \frac{h}{800} + 205.5$$

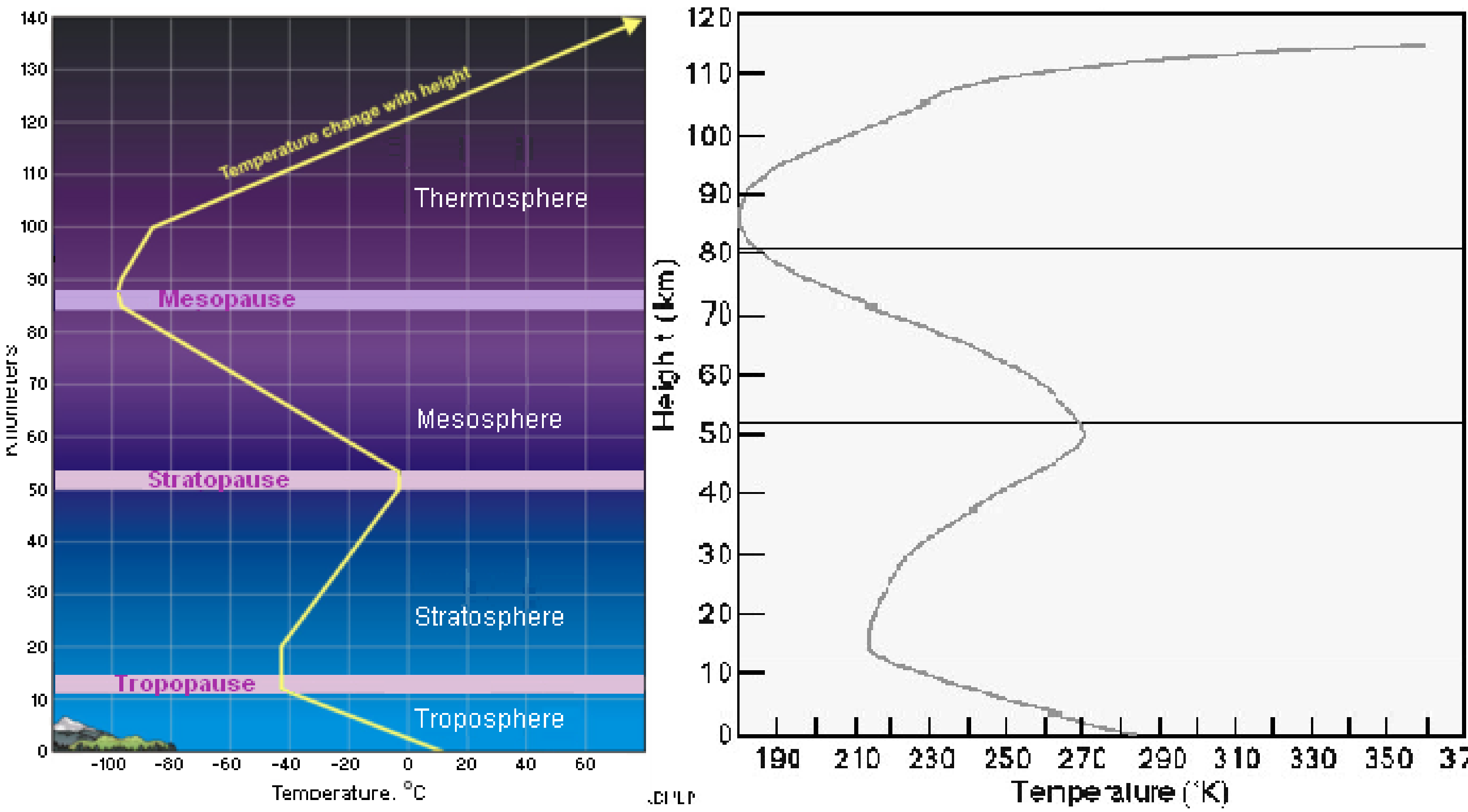

Figure 3: Graphs showing the relationship between altitude (in kilometres) and temperature (in Kelvin and Celsius).

Solution

The maximum altitude will be where the air displaced by our balloon is of the same mass as our balloon system, thus being neutrally buoyant.

If the air behaves as an ideal gas it obeys the equation

$$\frac{PV}{RT} = n$$

As

$$n \times M_r = M$$

$$\frac{PVMr}{RT} = M(ingrams)$$

Therefore

$$\frac{PVMr}{RT} = 34,800$$

Substituting our functions in terms of altitude:

$$\frac{P_o e^{-(\frac{h}{h_o})} \times \frac{4}{5}\pi r^3 \times 29}{8.31 \times \frac{h+164400}{800}}$$

$$\frac{10^5 e^{-(\frac{h}{7000})} \times \frac{4}{5}\pi 7.5^3 \times 29}{8.31 \times \frac{h+164400}{800}}$$

$$\frac{2.96 \times 10^9 e^{-(\frac{h}{7000})}}{h + 164400} = 34,800$$

Solving for h gives: $h_{max} = 26km$

The maximum altitude reachable by the balloon is 26km above sea level.

Possible Improvements

The model can be fine-tuned by adjusting several factors, i.e., adding a model of humidity variation in relation to altitude, modeling the divots in the balloons shell, or taking the balloon's thickness into account for internal and external volumes. All of these will enhance the model's accuracy. Incorporating a model of how internal pressures cause tensile stresses and whether the canvas can withstand these will show if the balloon can survive at height.

Is it possible to control the balloons altitude with an air pump?

An air pump attached to the shell of the balloon could be used to either release helium from the balloon in a controlled manner or to pump air into the balloon. Doing either would result in decreasing the balloons buoyancy allowing for controlled descents. Releasing helium will reduce the volume of the balloon, thus raising its mean density due to its constant dry mass, M_o. Pumping in air will increase the balloons mass whilst maintaining the same volume. This would require a high power pump to force air into the balloon, increasing the internal pressure and therefore stress on the shell. Both options allow for the balloon's altitude to be gradually decreased. This allows for data to be gathered at a range of heights and for low impact landings to be made, reducing the chance of damage to equipment on board the aerostat. Pumping air in may be seen as a more economical option with current crude helium prices; filling the balloon for take-off would cost approximately 346 US dollars. Pumping in air for landings would potentially allow for the helium to be recovered and recycled. This is assuming that a pump strong enough to overcome the internal pressures and able to add enough air for a landing, without being too expensive or heavy can be resourced. The air-helium mix could then be separated assuming the canvas can withstand the extra tensile stress. If not an alternative option is to gradually release the helium in a controlled manner.

Sources

- NASAs Scientific Balloon Programme Office
- RAVEN AEROSTAR
- Armed services technical information agency unclassified research reports
- WWW.spaceref.com
- Wikipedia
- The Engineering Toolbox
- Wolfram Alpha
- Desmos Graphical Calculator

Appendix

Altitude Above Sea Level		Absolute Barometer		Absolute Atmospheric Pressure		
feet	meters	inches Hg	mm Hg	psia	kg/cm^2	kPa
-5000	-1524	35.7	908	17.5	1.23	121
-4500	-1372	35.1	892	17.2	1.21	119
-4000	-1219	34.5	876	16.9	1.19	117
-3500	-1067	33.9	861	16.6	1.17	115
-3000	-914	33.3	846	16.4	1.15	113
-2500	-762	32.7	831	16.1	1.13	111
-2000	-610	32.1	816	15.8	1.11	109
-1500	-457	31.6	802	15.5	1.09	107
-1000	-305	31.0	788	15.2	1.07	105
-500	-152	30.5	774	15.0	1.05	103
0[1)]	0	29.9	760	14.7	1.03	101
500	152	29.4	746	14.4	1.01	99.5
1000	305	28.9	733	14.2	0.997	97.7
1500	457	28.3	720	13.9	0.979	96.0
2000	610	27.8	707	13.7	0.961	94.2
2500	762	27.3	694	13.4	0.943	92.5
3000	914	26.8	681	13.2	0.926	90.8
3500	1067	26.3	669	12.9	0.909	89.1
4000	1219	25.8	656	12.7	0.893	87.5
4500	1372	25.4	644	12.5	0.876	85.9
5000	1524	24.9	632	12.2	0.860	84.3
6000	1829	24.0	609	11.8	0.828	81.2
7000	2134	23.1	586	11.3	0.797	78.2
8000	2438	22.2	564	10.9	0.768	75.3
9000	2743	21.4	543	10.5	0.739	72.4
10000	3048	20.6	523	10.1	0.711	69.7
15000	4572	16.9	429	8.29	0.583	57.2
20000	6096	13.8	349	6.75	0.475	46.6
25000	7620	11.1	282	5.45	0.384	37.6
30000	9144	8.89	226	4.36	0.307	30.1
35000	10668	7.04	179	3.46	0.243	23.8
40000	12192	5.52	140	2.71	0.191	18.7
45000	13716	4.28	109	2.10	0.148	14.5
50000	15240	3.27	83	1.61	0.113	11.1

Figure 4: Engineering toolbox's table of pressures.