Let K be a compact set in a metric space (X, d). Suppose $\mathcal{F} = \{U_\alpha\}_{\alpha \in A}$ is an open cover of K, then there exists a positive number λ so that for every $p \in K$ the open ball $B(p, \lambda)$ is contained in one of the open sets of $\mathcal{F}$.

Proof. Since $K \subset \bigcup_{\alpha \in A} U_\alpha$, for each point p in K there is a positive number $2\varepsilon(p)$ so that the ball $B(p, 2\varepsilon(p))$ is contained in one of the open sets of $\mathcal{F}$. Clearly $\{B(p, 2\varepsilon(p)\}_{p \in K}$ forms an open cover of K, and so by compactness this admits a finite refinement □