Exterior Angle Theorem

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Write a (very brief) introduction here. You should include the name of the original presenter of the result, and (if appropriate) the name of the person who posed the question or conjecture originally. Also include the names of any other helpful persons to your write-up, the presentation, or the solution itself.

Include some narrative that will connect this result to others or give some of the background on the questions or conjectures the result answers. (This will be easier in some cases than others; don't worry too much about being eloquent here . . . focus on the art of proof-writing first.)

Proposition I.15. *The exterior angle of a triangle is greater than either of the interior and opposite angles.*

Proof. Let ABC be a triangle, and extend segment BC to point D by Euclid's Postulate 2. We wish to show that exterior angle ACD is greater than each of the interior angles CBA and BAC.

Construct E to be the midpoint of segment AC, by Euclid I.10, and construct the segment BE by Euclid's Postulate 1 and extend BE beyond point E by Euclid's Postulate 2. Using Euclid I.3, construct the point F on the extended segment BE such that BE is congruent to EF. Construct the segment CF by Euclid's Postulate 1.

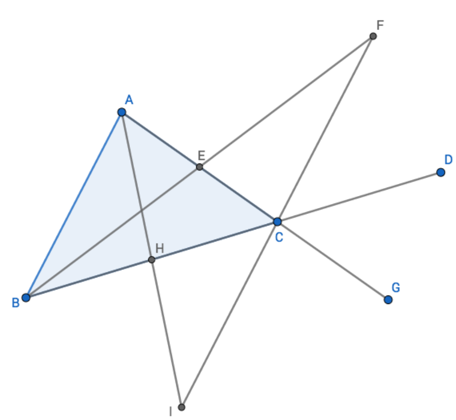

Figure 15: $\overline{AE} \cong \overline{EC}$, $\overline{BE} \cong \overline{EF}$, $\overline{BH} \cong \overline{HC}$, and $\overline{AH} \cong \overline{HI}$.

Segments AE and EC are congruent by construction; as are segments BE and EF. Since angles AEB and CEF are vertical angles, they are also congruent by Euclid I.15.

Thus, by Euclid I.4 (SAS congruence) triangles BAE and CEF are congruent and angle EAB is congruent to angle ECF.

Angle ECD is greater than angle ECF by Euclid's Common Notion 5. Therefore, angle EAB is greater than angle ECD.

This shows that the exterior angle, namely ACD, is greater than one of the interior opposite angles, namely CAB.

To show that the exterior angle is also greater than the remaining opposite interior angle, ABC, similarly extend segment AC to G, bisect segment BC at H, construct the segment AH, and extend to I such that segments AH and HI are congruent. The analogous argument shows that triangles ABH and ICH are congruent, making angles HBA and HCI congruent. But, angles ACD and HCG are congruent, and angle HCG is greater than HCI. This shows that the exterior angle ACD is greater than the opposite interior angle ABC as desired. $\square$

If there is any narrative that makes sense here, or closing remarks, feel free to include something. ☺