Homework Assignment MAT214 Template Jason Gumaer

We need to prove: $\forall n, m \in \mathbb{Z}$, if n - m is even, then $n^3 - m^3$ is even.

Proof: Assume n and m are integers such that n - m is even.

Since n - m is even, by the definition of an even number, $\exists k \in \mathbb{Z} : n - m = 2k$.

Then,
$$n^3 - m^3 = (n - m)(n^2 + nm + m^2)$$
 by difference of perfect cubes,
= $2k(n^2 + nm + m^2)$ by substitution,
= $2(n^2k + nmk + m^2k)$ by distribution.

We know $n, m, k \in \mathbb{Z}$. The integers are closed under addition, multiplication, and when integers are squared. So, we know $n^2k + nmk + m^2k$ is an integer under closure. Thus, $\exists j \in \mathbb{Z} : n^2k + nmk + m^2k = j$.

Now, we can say $n^3 - m^3 = 2j$ by substitution. This is the definition of an even integer. Therefore, $n^3 - m^3$ is even.

QED

Here are a few other items that could be useful when writing in $\[\] \[mathbb{LT}\[mathbb{E}\[mathbb{X}\].$

First, if you noticed above, we are using an inline mathemode every time we have a set of \$. This automatically formats the typing to look a specific way. Most symbols begin with a \followed by the name of the symbol. There are numerous tables you can look up for these.

For instance, if I wanted to write an integral in mathemode, it looks like this: $\int_a^b f(x) dx$. If you notice, the integral looks a little small. This is so everything fits within the margin of the line. If we want to use the inline mathemode but make all the symbols the proper size, we can write it like this: $\int_a^b f(x) dx$. If you notice, some of the notation is larger, but the paragraph now needs more spacing between the lines.

We can also use a dedicated mathemode that is not for writing inline. This creates a full break, centers the code, and automatically makes all the symbols the proper size.

$$\int_{a}^{b} f(x) \, dx$$

Now that mode is just for writing a single line of mathematics and it cannot be used for writing multiple lines. For that we either use the equation operation with begin and end or the align operation like in the proof above.