THEOREM 2.1

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ABSTRACT. We prove theorem 2.1 using the method of proof by way of contradiction. This theorem states that for any set A, that in fact the empty set is a subset of A, that is $\emptyset \subset A$.

We first start with a discussion of subsets.

Definition 1. *Let A and B be sets. We say A is a subset of B if every element in A is also an element of B and we write $A \subset B$. This can also be written as*

$$(A \subset B) \leftrightarrow \forall x (x \in A \rightarrow x \in B).$$

Notice that for sets A and B, if $A \not\subset B$, then there exists an element x such that $x \in A$ and $x \notin B$. That is,

$$(A \not\subset B) \leftrightarrow \exists x (x \in A \wedge x \notin B).$$

Example 1. *Let $A = \{1, 2, 3, 4, 5\}$, $B = \{1, 2\}$ and $C = \{1, 7\}$. We can see that every element in B is an element of A. Further, we can see that C contains an element, namely 7, which is not in A. Thus, $B \subset A$ and $C \not\subset A$.*

We now prove theorem 2.1.

Theorem (2.1)**.** *For any set A, $\emptyset \subset A$.*

Proof. By way of contradiction, suppose that the theorem fails. Let A be a set such that $\emptyset \not\subset A$. From the above discussion, we can see that there exists an element x such that $x \in \emptyset$ and $x \notin A$. Let x be such an element. Since the emptyset has no elements, then $x \notin \emptyset$. Thus, we have that $x \in \emptyset$ and $x \notin \emptyset$. This contradiction proves that the theorem is true. □